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ORIGINAL RESEARCH PAPER

Spectral modal modeling by FEM of reinforced concrete framed buildings irregular in elevation

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ABSTRACT

The irregular buildings constitute a large part of urban infrastructure and they are currently adopted in many structures for architectural or esthetic reasons. In contrast, the behavior of these buildings during an earthquake generates a detrimental effect on their regularity in elevation which leads to the total collapse of these structures.

The objective of this work is essentially to model reinforced concrete framed buildings irregular in elevation subjected to seismic loads by the Finite Element Method (FEM). This modeling aims to evaluate several parameters: displacements, inter-storey drifts and rigidities, using two dynamic calculation methods; one modal and the other spectral modal. The latter is widely used by engineers.

For this purpose, a detailed study of the frames which have several setbacks in elevation is carried out to validate the correct functioning of our FEM calculation code in both cases of modal and modal spectral analyses. The performance, accuracy and robustness of the FEM calculation code produced in this study is shown by the good correlation of the obtained results for the treated frames with those obtained using the ETABS software.

KEYWORDS

modeling, finite element method, elevation irregularity, reinforced concrete frame, modal analysis, spectral modal analysis, dynamic calculation

1. INTRODUCTION

With the current evolution, an increase in the demand for sustainable development has been recorded, the people are interested to stay in urban areas rather than in rural areas. In these urban areas, the structures are usually in the form of buildings which have a safe behavior under the effect of the seismic action which depends on several factors: rigidity, an adequate lateral resistance, ductility as well as simple and regular configurations [1]. There is much less risk to buildings with regular geometry and uniformly distributed mass and stiffness in plan and elevation than with irregular configurations [2]. But nowadays, the need and demand of the latest generation have made it inevitable for engineers to plan buildings with irregular configurations [3].

Furthermore, in this kind of buildings, there are different types of irregularities depending on their location and extent, but mainly they are classified into two types, in plan and elevation [3]. Irregular structures in the plan are those in which the seismic response is not only in translation but also in torsion and is the result of the rigidity and/or the eccentricity of the mass in the structure [4]. While irregular structures in elevation are those in which mass or stiffness or geometric regularity are not uniform throughout the structure [4]. These structures present weaknesses that can be produced between adjacent storeys by discontinuities of rigidity, strength or mass [5]. Such discontinuities between floors are often related to abrupt variations in the geometry of the frame along with the height. There are many examples [6, 7] of building failures in past earthquakes due to discontinuities in elevation.

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In reality, structures are often irregular because the perfect regularity is an idealization that rarely occurs [8]. Therefore, the configurations irregular in elevation have often been recognized as one of the main causes of failure in earthquakes [5].

The seismic regulations, such as the Algerian Earthquake Regulations 1999 Version 2003 (RPA 99V2003) [9] and the Eurocode 8 [10], recommended the use of spectral modal dynamic analysis or temporal dynamic analysis as the preferred calculation methods to assess the seismic response of irregular buildings.

Several investigations have been carried out to study the seismic behavior of irregular structures in elevation. The first studies on the seismic behavior of buildings characterized by discontinuities along with the height in terms of mass, rigidity and resistance date back to the 1980s [11, 12], starting with the work of Moehle and Asce (1984) [11]; Costa et al. (1988) [13]; Shahrouz and Moehle (1990) [14]; Wood (1992) [15]; Wong and Tso (1994) [16]; Cassis and Cornejo (1996) [17]; Valmundsson and Nau (1997) [18]; Magliulo et al. (2002) [19]; Romão et al. (2004) [1]; they found differences in the response of irregular and regular frames. Among the most notable differences, they noted increases in ductility demands at the storey where the setback in elevation is located and also at the storeys immediately above.

The seismic efficiency of vertically irregular structures has recently been studied in many previous studies (Karavasilis et al. (2008) [20]; Sarkar et al. (2010) [21]; Rajeev and Tesfamariam (2012) [22]; Varadharajan et al. (2013) [23]; Pirizadeh and Shakib (2013) [24]; Roy and Mahato (2013) [25]; Hamdani (2015) [26]).

More recently, Darshale and Shelke (2016) [27], Bhosale et al. (2017) [28] , Shelke et Ansari (2017) [29], Siva Naveen et al. (2019) [30], Etli et Güneyisi (2020) [31], examined studies on the analysis of irregular structures under seismic loads, they concluded that the irregular distribution of mass, stiffness and vertical geometric irregularity showed a different response from the regular one.

In addition, several techniques have been used in the numerical simulation of reinforced concrete frames buildings. Among them, we have the most used methods in the field of engineering, we can recall: the finite differences method (FDM) [32] for diffusion problems (heat transfer, fluid flow, etc.), the boundary element method (BEM) [33] for the problems interfaces and interactions; the finite volume method (FVM) [34] and the Finite Element Method (FEM) [35]. In this work, we will choose this last method as a method of solving problems of irregular structures in elevation. The concept of this method is used for numerical analysis wherein the model to be studied is divided into elements known as finite elements and each element's response is expressed in terms of a finite number of degrees of freedom [35]. Moreover, the finite element method is proving to be a very powerful tool for the analysis of structures which remain continuous, subjected to violent accidental aggression such as an earthquake [36].

Various researchers have conducted studies on numerical modeling using FEM to study regular steel gantries under seismic excitation such as Ozturk and Catal [37], Sekulonic et al. [38], Shousuke and Yasuhiro [39] et Sharbane and Niraj Kumar [40].

In the present work, we have modeled the reinforced concrete framed buildings irregular in elevation subjected to a seismic excitation by the FEM. To find the displacements, two dynamic calculation methods were used: the method of modal analysis and the method of spectral modal analysis. The numerical modeling of these irregular frames represents the originality of this study compared to the work mentioned above.

2. THE IRREGULARITY IN ELEVATION OF REINFORCED CONCRETE STRUCTURES

The seismic action is an accidental action which is defined in the Algerian Earthquake Regulations [9]. Indeed, to avoid the damages resulting from this seismic action, a judicious earthquake-resistant design must be carried out which ensures adequate seismic behavior. For example, discontinuities in stiffness and strength should be avoided, which should ideally be distributed evenly over the height of the structure [18, 41].

However, in practice, reinforced concrete frame buildings are generally found to be irregular in shape whose irregularity is characterized by a setback in elevation (Fig. 1).

Fig. 1. Irregular buildings in elevation

Fig. 2. Damage in re-entrant angles in the vertical plane due to differential oscillations: (a) Boumerdes earthquake, Algeria 2003 [42], (b) Kobe earthquake, Japan 1995 [43]

This vertical irregularity is one of the main factors of structural failures [41]. The changes in height of stiffness and mass make the dynamic characteristics of these buildings different from those of the normal or regular building (Fig. 2).

3. DYNAMIC MODELING BY FEM OF IRREGULAR FRAMES

To model the frame buildings in reinforced concrete, the finite element approximation is given as follows [35]:

$$u(x,\, t) = \sum_{i \in n} N_i(x) q_i(t) \tag{1}$$

where $u(x,\, t)$ represents the field of approximate displacements, $N_i(x)$ represents the standard interpolation functions and $q_i(t)$ the vector of nodal displacements, n the set of nodes of the discretized domain [35]. The use of the Rayleigh-Ritz method allows us to write the linear matrix equation describing the behavior of a dynamic system:

$$[M]\{\ddot{q}(t)\} + [C]\{\dot{q}(t)\} + [K]\{q(t)\} = \{F(t)\} \tag{2}$$

with:

$$[M] = \int_{\Omega} \rho [N]^T [N] d\Omega \tag{3}$$

$$[K] = \int_{\Omega} [B]^T [D][B] d\Omega \tag{4}$$

where ρ is the density, $[B]$ is the strain matrix derived from the interpolation functions and $[D]$ the elasticity matrix. The matrices $[M]$, $[C]$ and $[K]$ are respectively the matrices of mass, of damping and elastic rigidity. The vector $\{F\}$ is the vector of dynamic excitations.

The integration of Eqs. (3) and (4) on the domain Ω brings us back to an assembly of elementary matrices. The boundary conditions are imposed before solving the dynamic system (Eq. (2)). Finally, the damping matrix $[C]$ is expressed as a linear combination of the two matrices $[K]$ and $[M]$, using the Rayleigh damping, which is written as:

$$[C] = \alpha[M] + \beta[K] \tag{5}$$

where α and β are the Rayleigh coefficients.

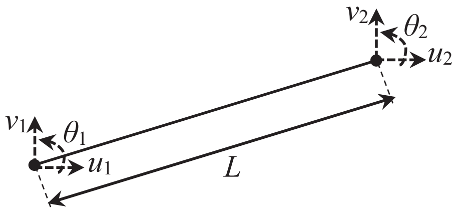

Fig. 3. Bar element in the plane with six degrees of freedom [35]

To numerically model the frames and establish the matrices already mentioned we used a linear element (bar) that has two nodes with three degrees of freedom per node [35] (Fig. 3).

The stiffness and mass matrices are calculated by analytical integration for the bending bar element composed as follows [44]:

$$[K^e] = \begin{bmatrix} \frac{EA}{L} & 0 & 0 & -\frac{EA}{L} & 0 & 0 \\ & \frac{12EI}{L^3} & \frac{6EI}{L^2} & 0 & -\frac{12EI}{L^3} & \frac{6EI}{L^2} \\ & & \frac{4EI}{L} & 0 & -\frac{6EI}{L^2} & \frac{2EI}{L} \\ & & & \frac{EA}{L} & 0 & 0 \\ & & & & \frac{12EI}{L^3} & -\frac{6EI}{L^2} \\ Sym. & & & & & \frac{4EI}{L} \end{bmatrix} \tag{6}$$

$$[M^e] = \frac{\rho AL}{2} \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix},$$

For how to calculate the displacement, there are several dynamic methods. In this study, we have chosen the method of modal and modal spectral analysis because they are the most recommended by the RPA 99V2003 [9] in the case of irregular structures.

3.1. Modal analysis

The solution of displacement $u(t)$ of a linear dynamic problem can be approximated by its decomposition on a frustoconical basis of eigenmodes Φ_j [45, 46]. Then it can be expressed as follows:

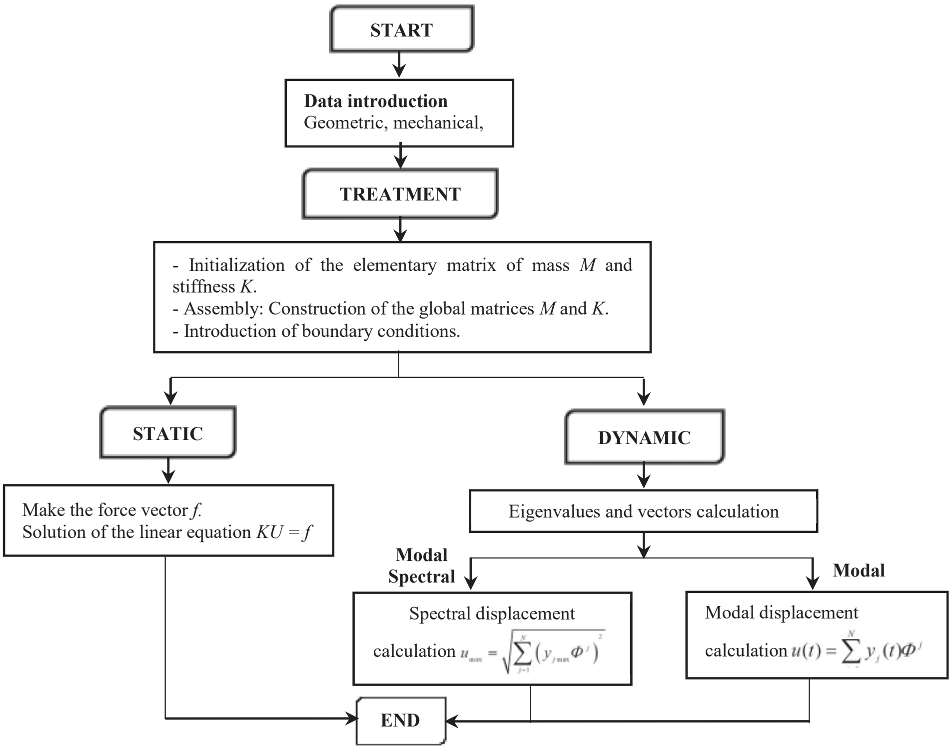

Fig. 4. Flowchart of the digital model

$$u(t) = \sum_{j=1}^{N} y_j(t) \Phi^j \tag{7}$$

where $y_j(t)$ are the generalized displacements which are evaluated by the Duhamel integral [47, 48].

The pulsations ω_j and the eigenmodes Φ_j are determined from the following relation [47, 48]:

$$\det\left[K - \omega_j^2 M\right] = 0,\ et \left[K - \omega_j^2 M\right] \Phi^j = 0\,. \tag{8}$$

3.2. Spectral modal analysis

Spectral modal analysis is a dynamic method based on the modal analysis method. Once the periods of oscillation of the structures have been established (modal analysis), one reads the spectral acceleration assumed to be a maximum response (spectral analysis). This last method uses the response spectrum which is a tool to estimate the response of a seismic building [49].

Due to its simplicity, this method is frequently used by most computer structural analysis engineers dealing with seismic design [50].

Thus, for a given earthquake, the maximum displacement of the structure is defined as the quadratic combination of modal displacements [50, 51]:

$$u_{\max} = \sqrt{\sum_{j=1}^{N} \left(y_{j\max} \Phi^j\right)^2}. \tag{9}$$

All the formulas explained above have been programmed in MATLAB [52] and organized in the form of the flowchart presented in Fig. 4.

4. NUMERICAL APPLICATION

To model and evaluate the irregular systems in elevation adopted in this work, a framed building in reinforced concrete was chosen. It is a structure of 06 levels, selected from the literature [8, 30]. This structure is made up of 3 spans of 5.00 m each and a height of 4.00 m for the first level and 3.00 m for the other levels (Fig. 5 (a)). We modified this structure to study the effect of irregularity in elevation by removing some structural elements from each level (See Fig. 5 (b), (c), (d), (e) et (f)).

The frames were designed in accordance with the provisions of the calculation code for Reinforced Concrete in Limit States BAEL91 [53] and the RPA 99V2003 [9]. The data of the frame, in particular, the sections of the beams and the columns are of 40 cm × 40 cm.

In the design procedure, the permanent and operating loads are distributed evenly over the floor and their values are:

for standard flooring : $G = 5.10\ \mathrm{KN/m^2}$ and $Q = 1.50\ \mathrm{KN/m^2}$.
for terrace flooring : $G = 6.20\ \mathrm{KN/m^2}$ and $Q = 1.00\ \mathrm{KN/m^2}$.

The modal and modal spectral analysis methods are used to study the seismic response of these frames which are excited by the same acceleration and spectral response load

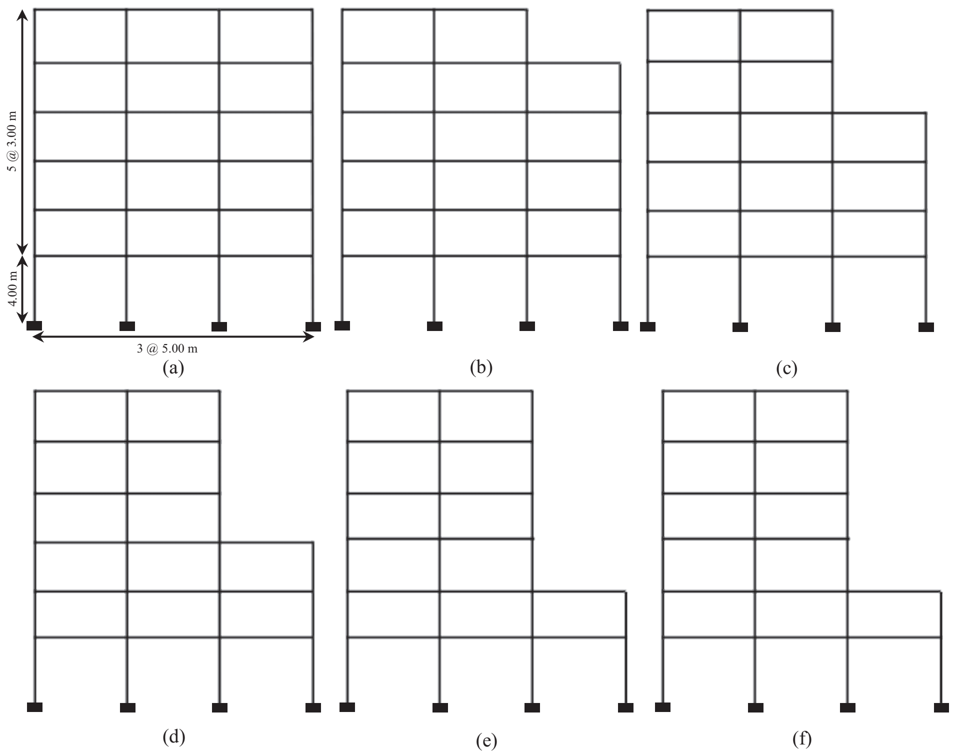

Fig. 5. The geometric configuration of the studied structure: (a) Regular, (b) Irregular N° 01, (c) Irregular N° 02, (d) Irregular N° 03, (e) Irregular N° 04, (f) Irregular N° 05

recorded during the Boumerdes earthquake (Algeria) in 2003 and which are presented in Fig. 6 (a and b).

The mechanical characteristics of the material are Young's modulus $E = 32164195$ KPa, the Poisson's ratio is $\nu = 0.18$ and the density $\rho = 2{,}400$ Kg/m^3, the damping coefficient is taken equal to 0.05.

5. RESULTS AND DISCUSSIONS

The theory presented previously has been programmed and implemented with our cases of irregular frames. And to show the performance of the developed program, a comparison is made with the ETABS software [54].

The obtained results in regular and irregular structures in terms of natural periods, storey displacement profiles, interstorey drift and finally the rigidity of each storey are presented.

5.1. Preliminary results

The results in terms of natural periods obtained by our calculation code FEM and ETABS [54] for the different studied models are shown in Table 1. It is clear that the

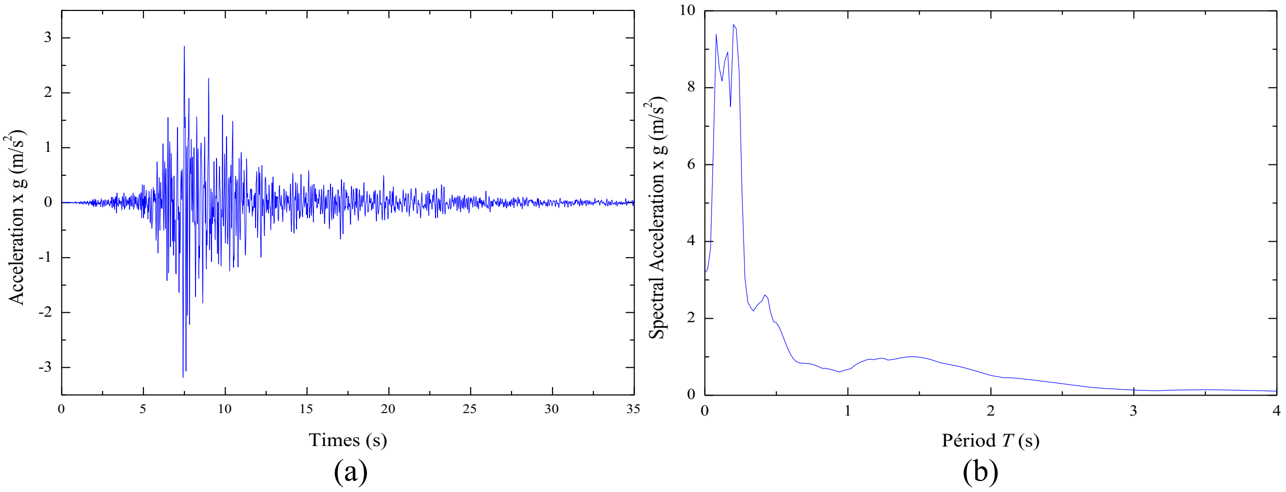

Fig. 6. Boumerdes earthquake: (a) Accelerogram, (b) Response spectrum

Table 1. The natural periods of the regular and irregular structure

			Case of the structure					
				Irregular				
Modes			Regular	N° 01	N° 02	N° 03	N° 04	N° 05
Mode 1	Periods (s)	Our code FEM	1.118	1.065	1.021	0.998	1.004	1.035
		ETABS	1.126	1.073	1.029	1.005	1.011	1.042
Mode 2		Our code FEM	0.357	0.343	0.354	0.373	0.366	0.345
		ETABS	0.359	0.346	0.357	0.375	0.369	0.348
Mode 3		Our code FEM	0.198	0.196	0.205	0.194	0.198	0.201
		ETABS	0.199	0.198	0.207	0.195	0.199	0.203

results acquired by our calculation code FEM are in good correspondence with those obtained by ETABS.

We can also notice that the natural periods for the regular structure are higher than those where the structure is irregular. Which means that the irregularity in elevation made the structure more flexible. Also, it is noted from the values of the natural periods of the irregular structures that a reduction of these values occurred by changing the type of structure. For the irregular structure N° 05, the first mode reaches 1.035.

5.2. Modal displacement profiles

The frames are excited by the Boumerdes earthquake 2003. The results of modal displacements in the X direction obtained by our code FEM and ETABS [54] are presented in Fig. 7.

These displacement curves are represented for the most stressed node of each frame case. Remarkably, the obtained results by our code FEM are very close to those obtained by ETABS. So, we can conclude that the obtained results in terms of modal displacements are satisfactory and promising.

We can also notice from the obtained curves that the displacement has increased considering the irregularity in elevation. This confirms that the irregular structure is dangerous because the setbacks in elevation can lead to serious damage, especially in cases of major earthquakes.

5.3. Profiles of spectral modal displacements

The frames are solicited by the response spectrum of the same Boumerdes earthquake (Algeria) 2003. The displacements obtained by our code FEM are presented with the max modal displacements and those obtained by ETABS [54] and this for each level of the regular and irregular frames N° 01, 02, 03, 04 and 05 (See Fig. 8).

We noted initially, a good correspondence between the results of spectral displacements and the max modal displacements obtained by our code FEM as well as those obtained by ETABS, and that the displacements increase according to the level of the storey.

Secondly, we can conclude that the displacements calculated by the spectral modal analysis are greater than those determined by the modal analysis for the same type of structure and number of the storeys, except for special cases. This is due to the principle of the spectral modal analysis method which uses the max accelerations.

To better understand the shape of the spectral displacements we have presented the displacement of the setback point of each type of structure (Fig. 9). It is clear that this setback in elevation will influence the results of displacements, this is perhaps due to the reduction in the rigidity of the structures.

5.4. Inter-storey drift

The calculation of inter-storey drifts is carried out taking into account the obtained results by our calculation code FEM using spectral modal analysis and which are presented in Fig. 10. The variations in elevation of these displacements are compared with an inter-storey drift of 1% of the storey height, which is required by the Algerian Earthquake Regulations RPA 99V2003 [9].

We can notice that the computed relative displacements do not exceed the displacement threshold admitted by the RPA 99V2003 [9]. It should be noted that the inter-storey drift in the case where the structure is irregular decreases compared to the regular structure.

5.5. Storeys Stiffnesses

Figure 11 shows the rigidity shapes of each storey by changing the frame type (regular – irregular). It should be noted that these shapes are reached using the obtained results by our calculation code FEM. We can see from the obtained curves that the stiffness increases with the storey level and decreases by changing the irregularity of the frame.

6. CONCLUSION

This study presents a computational procedure to model numerically the structures that present a setback in elevation using the FEM. The dynamic results are obtained by the modal and modal spectral analysis method. After a comparison between the obtained results by our computer code and those obtained by ETABS we concluded that there is a

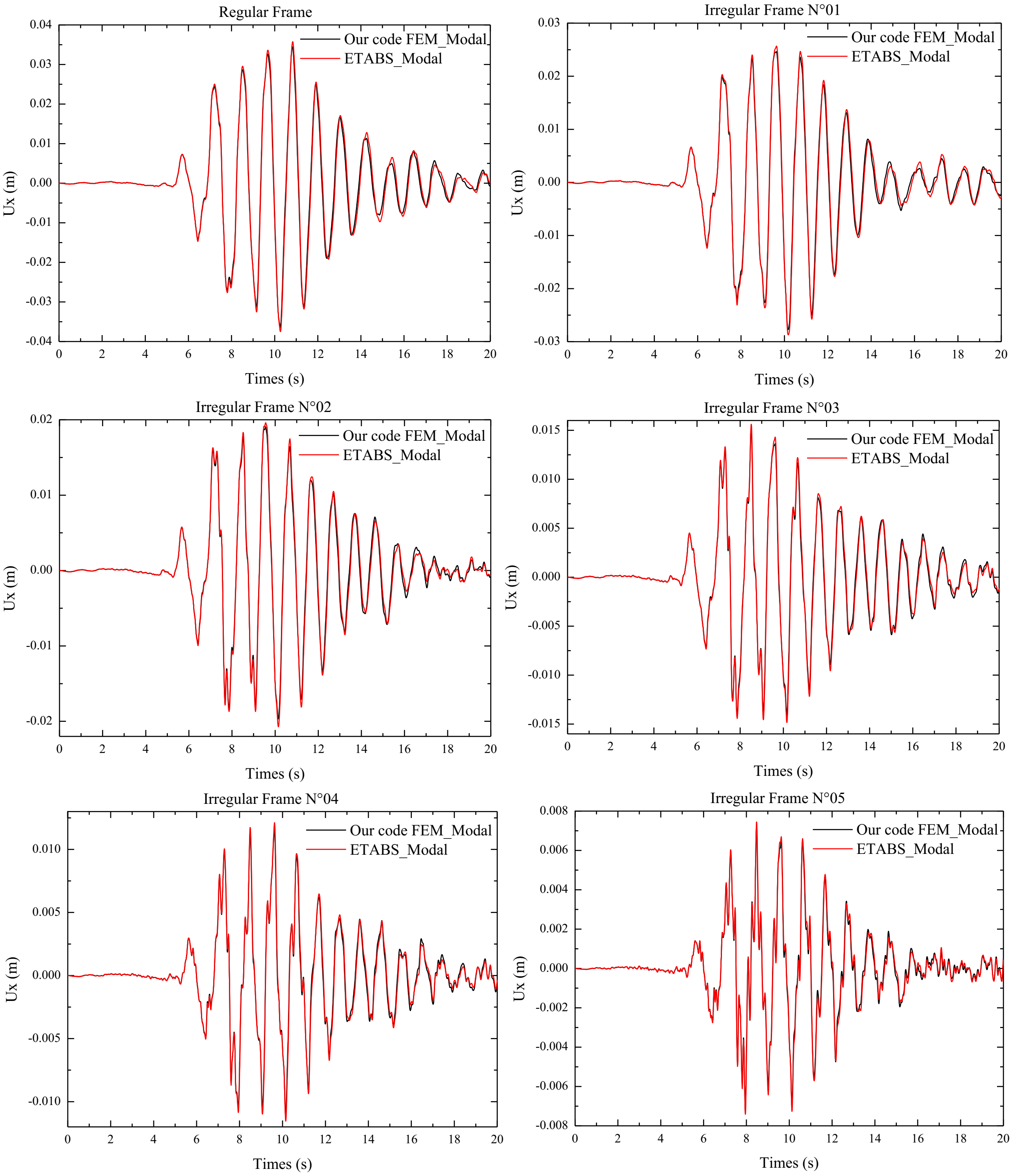

Fig. 7. The modal dynamic displacements Ux for the regular and irregular frame N° 01, 02, 03, 04 and 05

good correlation between these results for several configurations processed. So, we may say that our code gives us full satisfaction in terms of the accuracy and richness of the results obtained.

Also, we have concluded some remarks which are summarized as follows:

1. Given the irregularity in elevation, the modal displacements have increased.
2. With the exception of special cases, the displacements measured by the spectral modal analysis are greater than those determined by the modal analysis for the same structure form and storey level.

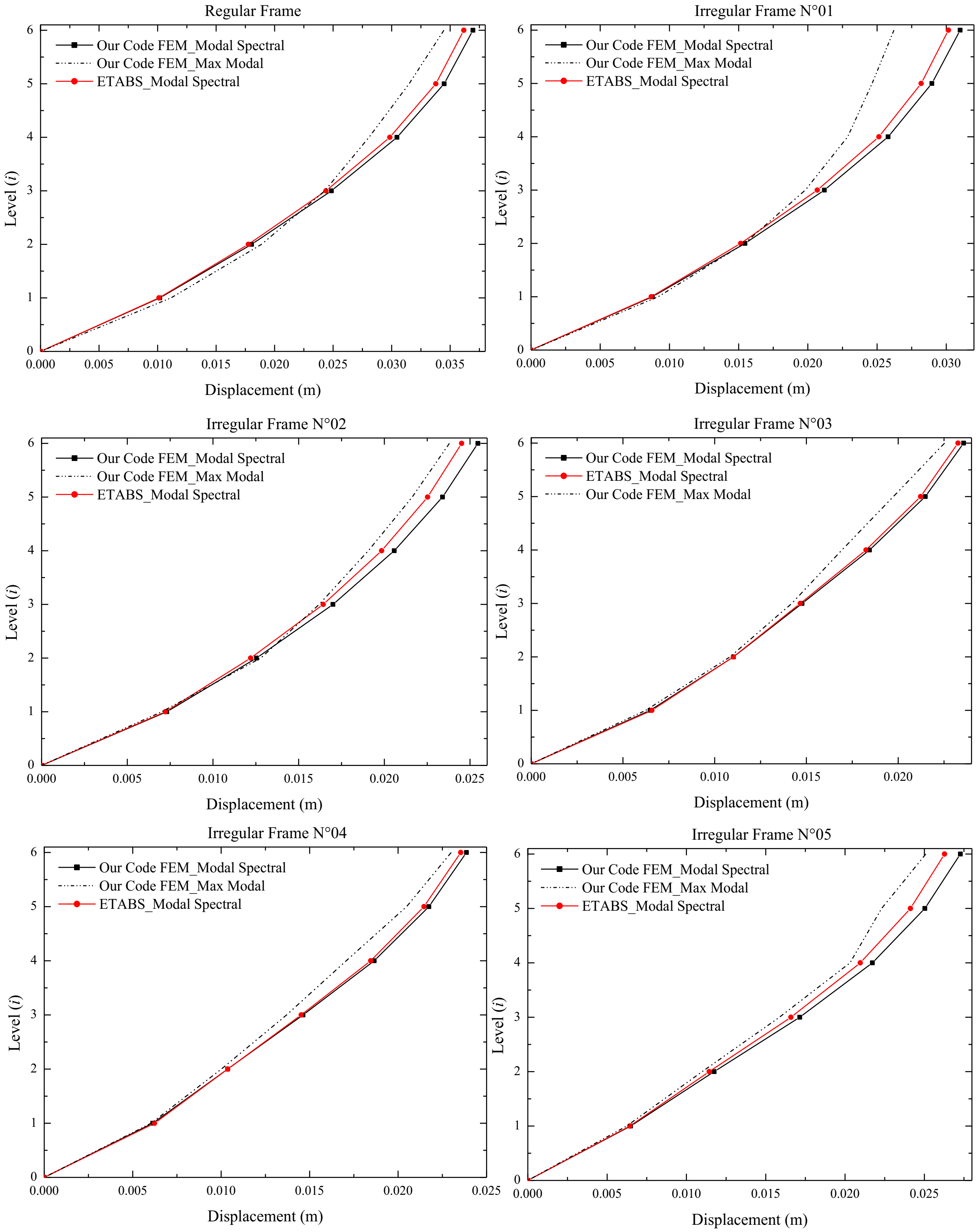

Fig. 8. The spectral modal dynamic displacements for the regular and irregular frame N° 01, 02, 03, 04 and 05

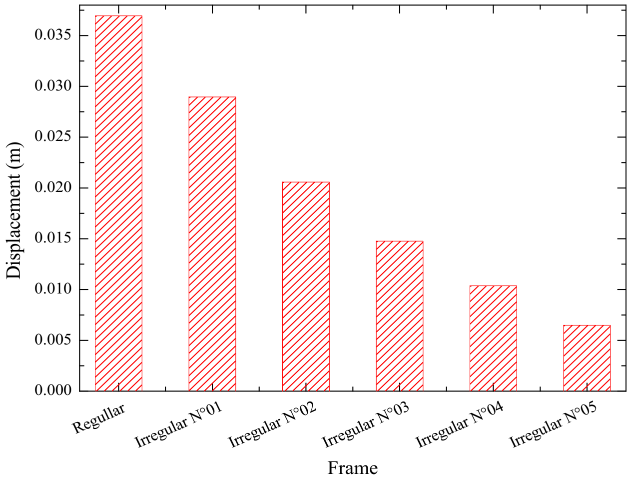

Fig. 9. The spectral displacement of the setback point of each type of regular and irregular frame N° 01, 02, 03, 04 and 05

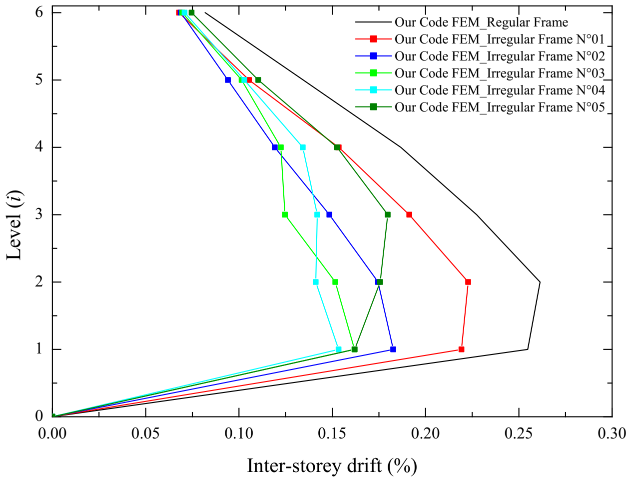

Fig. 10. Inter-storey drifts of the regular and irregular structure N° 01, 02, 03, 04 and 05

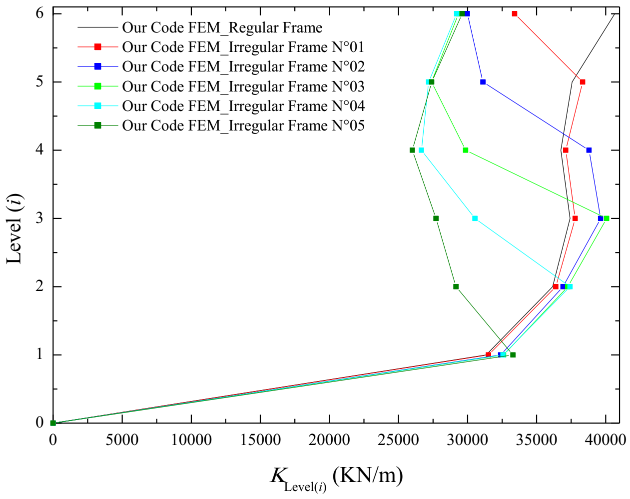

Fig. 11. The Storeys Stiffnesses of the regular and irregular structure N° 01, 02, 03, 04 and 05

3. The setbacks in elevation influence the results of displacements – this is perhaps due to the reduction in the rigidity of structures.
4. Compared with the regular structure, the inter-storey drift decreases in the case of the irregular structure.
5. The rigidity increases with the storey level and decreases by changing the frame irregularity.

Therefore, from these results, we can conclude that we will have to be careful when using this type of irregular structures, especially in high seismicity areas because this type has caused enormous damage in past catastrophic earthquakes.

Finally, we can develop and extend this work in the future so that we could study the finite element modeling of reinforcement of irregular structures in elevation by another bracing system which would avoid their ruin and minimize rigidities.

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