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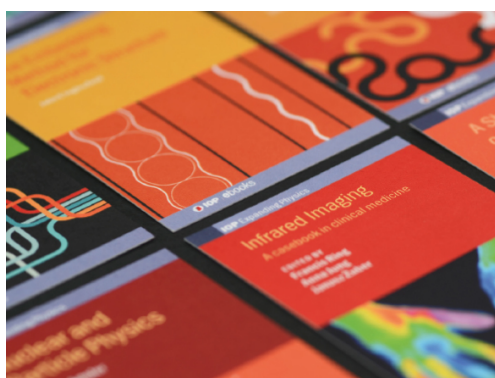
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Exotic shapes and clusterization of atomic nuclei

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Abstract. Shape isomers, including superdeformed and hyperdeformed states can be determined from the quasi-dynamical $SU(3)$ symmetry based on the Nilsson-model. We investigate the possible binary clusterizations of these shape isomers. The allowed cluster-configurations give a hint for the favourable reaction channels for populating these states. Our semimicroscopic approach is largely based on symmetry considerations, combined with energetic preference calculations. As illustrative examples some results for the ^{36}Ar , ^{40}Ca and ^{56}Ni nuclei are shown.

1. Introduction

The exotic shapes of atomic nuclei attract much attention recently both from the experimental and from the theoretical sides. Theoretically they are usually obtained from shell-model or mean-field calculations as the minima of the energy-surface. Here we discuss an alternative possibility of determining these states from the Nilsson-model, via their quasi-dynamical (or effective) $SU(3)$ symmetry [1, 2]. The study of the possible clusterizations of these exotic states are important not only for the better understanding of their structure, but also for predicting the appropriate reaction channels for their formation.

Some superdeformed bands in $N = Z$ nuclei were observed experimentally during the last decade. As for their hyperdeformed states are concerned, practically they belong to a domain for theoretical studies.

A remarkable example is that of the ^{36}Ar nucleus. The superdeformed shape of the ^{36}Ar nucleus was detected in 2000 [3], and some theoretical predictions were done on the hyperdeformed state from alpha-cluster [4], and binary cluster studies [5]. A recent study on molecular resonances in heavy-ion reactions seems to justify these predictions [6] not only from the respect of the moment of inertia of the band, but also concerning its clusterization, i.e. the preferred reaction channels for its population. Furthermore, the above-mentioned shape-calculation from Nilsson-model + quasi-dynamical symmetry gives a hyperdeformed state completely in line with the cluster models and with the experimental finding [7]. The superdeformed band of the ^{40}Ca nucleus has recently been observed experimentally [8] and the same band has been detected in an independent measurement [9]. For the ^{56}Ni two

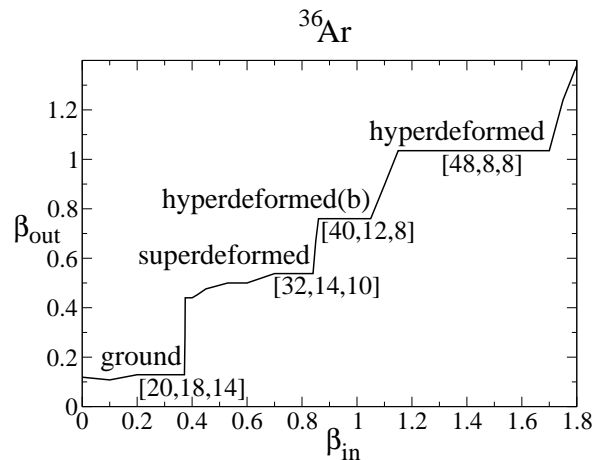


Figure 1. Quadrupole deformation of the ^{36}Ar nucleus from the Nilsson-model with the effective $U(3)$ quantum numbers at the plateaus.

deformed bands were observed in [10], and these bands were considered later on as examples of superdeformed bands [8].

2. Shape isomers

Nilsson-model calculations are very illuminating in searching for stable deformations and shape isomers [11]. Here we present the results of a new kind of calculation, based on the Nilsson-model. In particular, it consists in a self-consistency check with respect to the quadrupole deformation. It is done in terms of $U(3)$ symmetries in the following way. The $U(3)$ symmetry, which is an approximately good symmetry of light nuclei [12], is known to be uniquely related to the quadrupole shape [13]. Furthermore, when the real $U(3)$ symmetry breaks down, e.g. with increasing excitation energy (due to some symmetry breaking interactions, like spin-orbit, pairing, etc), a generalized version of it, called quasi-dynamical or effective $U(3)$ symmetry still survives [1]. The quasi-dynamical $U(3)$ quantum numbers can be obtained from Nilsson-calculations [14, 15]. Thus, the self-consistency calculation consists in the continuous variation of the quadrupole deformation, as an input for a Nilsson-calculation, and determination of the effective $U(3)$ quantum numbers or, from them, the corresponding β_{out} quadrupole deformation. The β_{out} as a function of the β_{in} is typically a "stair-like" function, as it can be seen in Figure 1. where the results for the input prolate deformations of the ^{36}Ar nucleus are shown. In this figure it is not the minima, rather the horizontal plateaus, which correspond to the stable shapes. They are stable in the sense, that they are insensitive to the small changes of the input parameter. Furthermore, at these deformations a good agreement between the input and output values can be observed. That is these deformations fulfill the self-consistency argument, to a reasonable approximation, between the input and output deformation-parameters.

For the ^{36}Ar nucleus this kind of calculation gives the ground state, of course, as the first stable shape. When proceeding towards larger deformation, the next one corresponds to the superdeformed shape, representing 4 nucleon excitation, being very much in line with the more recent Nilsson, as well as with the shell-model calculations [11, 3].

With increasing β -values two further stable plateaus appear, a less-pronounced one around 0.8 (β_{out}), and a very big one at cca 1.1. The first one coincides with the prolate state of [11]. The second one is in complete agreement with the prediction of the alpha-cluster model for the hyperdeformed state [4], as well as with the experimentally observed largely prolate band [6], as

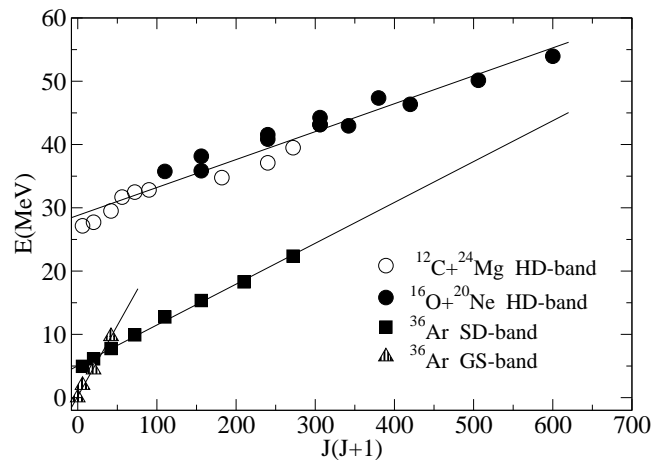


Figure 2. Excitation energy versus the angular momentum for the ground, superdeformed, and the recently observed hyperdeformed bands in the ^{36}Ar nucleus.

shown by Figure 2.

For ^{40}Ca and ^{56}Ni nuclei figures similar to Figure 1. have been obtained. For the ^{40}Ca the [20,20,20], [38,17,13] and [54,12,10] $U(3)$ quantum numbers have been found as characterizing the ground, superdeformed and hyperdeformed states, respectively. For the superdeformed state the $8\hbar\omega$ excitation is completely in line with the joint conclusion of the experimental and previous theoretical investigations [8, 16, 17]. For the ^{56}Ni the ground, superdeformed and hyperdeformed states have been identified as well. The superdeformed state is characterized by $4\hbar\omega$ excitation quanta similarly to other theoretical calculations [10] describing the experimental data.

For light nuclei, like ^{24}Mg and ^{28}Si , where detailed comparison could be made, the results of this self-consistency calculation are in very good agreement with that of the energy-minima calculations [18, 19].

3. Cluster structure

When studying the possible cluster structure of a specific nuclear state, like the ones of the previous section, one has to take into account the effects of two basic natural laws: the energy-minimum principle and the Pauli-exclusion principle. Fully microscopic cluster models are able to handle both of them in a completely satisfactory manner, however, their range of applicability is limited due to the large-scale calculations. We are interested in applying a semimicroscopic approach, which can be extended even to heavy nuclei and exotic clusterization [5, 20]. Symmetry-considerations can be helpful along this line, and exclude the Pauli-forbidden states without carrying out the antisymmetrization explicitly. For light nuclei even the real $SU(3)$ can lead us to useful results, but for the heavy ones we need to change to the more general quasi-dynamical symmetry. (This is applicable also for light nuclei, of course, as illustrated e.g. by the previous section. When the real $SU(3)$ is approximately valid, then it coincides with the quasi-dynamical one [15].)

There are two simple recipes, which are based on the microscopic picture, yet they are easy to apply systematically. These are the $U(3)$ selection rule [21], and Harvey's prescription [22]. In the case of a binary clusterization the $U(3)$ selection rule reads:

$$[n_1, n_2, n_3] = [n_1^{(1)}, n_2^{(1)}, n_3^{(1)}] \otimes [n_1^{(2)}, n_2^{(2)}, n_3^{(2)}] \otimes [n^{(R)}, 0, 0], \quad (1)$$

where $[n_1, n_2, n_3]$ is the set of $U(3)$ quantum numbers of the parent nucleus, the superscripts (i)

stand for the i th cluster, (R) indicates relative motion. A given clusterization is allowed if there is a matching between $[n_1, n_2, n_3]$ and the product representations. When a cluster configuration is forbidden, we can characterize its forbiddenness quantitatively [23]. The Harvey-prescription says that when two nuclei amalgamate, then only the number of quanta along the molecular axis can change, while numbers along the other two directions remain unchanged. Both of these rules applies the harmonic oscillator basis, thus there is a considerable similarities between them. However, they are not identical, rather, they are complementary to each other in a sense. Therefore, they should be applied in a combined way [19].

Since the $U(3)$ symmetry is related to the quadrupole deformation of the nucleus, the $U(3)$ selection rule can be interpreted as a consistency check of the quadrupole deformation, between the shell model (or collective model) states and the corresponding cluster states. It should also be mentioned that the $U(3)$ selection rule, which deals with the space-symmetry of the states, is always accompanied by a similar $U^{ST}(4)$ [24] selection rule for the spin-isospin degrees of freedom.

As for the energetic stability of the cluster-configuration is concerned, either binding-energy arguments [25], or double-folding calculation can be applied [26]. The criterium of maximal stability requires the largest value of the summed differences of the measured binding energies and the corresponding liquid drop values. In the dinuclear system model the potential energy U is minimized with respect to the mass asymmetry for each fixed charge asymmetry. Both methods indicate that the alpha-like (i.e. $N = Z = \text{even}$) configurations are energetically preferred. Thus we concentrate here on alpha-like binary clusterizations.

The superdeformed state of the ^{36}Ar nucleus shows a little shape-uncertainty. This is a joined conclusion of several theoretical studies. In our method this uncertainty is reflected by the appearance of close-lying, but not completely identical $U(3)$ representations. Therefore, we have considered the shapes corresponding to the $[32,14,10]$ effective quantum numbers, and to the $[32,16,8]$, which correspond to a simple harmonic oscillator state. The latter one allows (^4He , ^8Be , ^{12}C , ^{16}O)+core clusterizations, while the former one allows only the last two configurations (though the other ones are not very strongly forbidden either).

The hyperdeformed state on the other hand seems to have a very well-defined symmetry, and it allows only $^{24}\text{Mg}+^{12}\text{C}$ and $^{20}\text{Ne}+^{16}\text{O}$ binary configurations. (In these considerations the clusters are supposed to be in their ground intrinsic states. E.g. a $^{28}\text{Si}+^8\text{Be}$ configuration is allowed, too with the prolate shape of the ^{28}Si , but it is thought to be not the dominant component of the ground-state wavefunction.)

It is also remarkable that the $^{24}\text{Mg}+^{12}\text{C}$ clusterization is possible both in the ground-state, and in the superdeformed, as well as in the hyperdeformed state. The difference between these configurations is the relative orientation of the two deformed clusters with respect to the molecular axis. This observation could be made due to the facts that i) the Pauli-principle was taken into account, and ii) no oversimplifying model assumptions (e.g. spherically or cylindrically symmetric cluster shapes) were made.

Concerning the ^{40}Ca nucleus in its superdeformed state the allowed clusterizations are $^8\text{Be}+^{32}\text{S}$, $^{12}\text{C}+^{28}\text{Si}$ and $^{20}\text{Ne}+^{20}\text{Ne}$. In the hyperdeformed state of this nucleus the $^{16}\text{O}+^{24}\text{Mg}$ and the $^{20}\text{Ne}+^{20}\text{Ne}$ clusterizations are allowed.

We determined the possible binary alpha-like cluster-configurations for the super- and hyperdeformed states of ^{56}Ni as well. Similarly to the previous nuclei in its hyperdeformed state the more symmetric clusterizations are allowed, while in the superdeformed state asymmetric and symmetric clusterizations both appear.

4. Conclusions

With this contribution we wished to demonstrate that symmetry-considerations can be helpful in studying both the shape isomers and the possible clusterizations of atomic nuclei. They are

able to shed some light also on the interrelations of these two phenomena. In this respect they can be helpful in predicting the favourable reactions for the population of superdeformed and hyperdeformed states. The hyperdeformed state of the ^{36}Ar nucleus serves as a very interesting example. Its existence was first foreseen from alpha-cluster model [4], then binary-cluster studies predicted their population in $^{24}\text{Mg}+^{12}\text{C}$ and $^{20}\text{Ne}+^{16}\text{O}$ reactions [5]. Recent reaction studies [6] seem to justify these predictions, and structure-calculations [7] show that the same state is obtained from the Nilsson-model via the quasi-dynamical $SU(3)$ symmetry.

Acknowledgments

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