ON A CONJECTURE OF SCHÄFFER CONCERNING THE EQUATION $1^k + \cdots + x^k = y^n$

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Dedicated to N. Saradha on the occasion of her 60th birthday.

Abstract. We prove Schäffer's conjecture concerning the solutions of the equation in the title under certain assumptions on x, letting the other variables k, n, y be completely free. We also provide upper bounds for n under more moderate conditions. Finally, we give all solutions of the equation in the title for some concrete values of x. Our results rely on assertions describing the precise exponents of 2 and 3 appearing in the prime factorization of $S_k(x)$ and on the explicit solution of polynomial-exponential congruences.

1. Introduction

For positive integers k and x, write

$$S_k(x) = 1^k + \cdots + x^k$$

for the sum of the k-th powers of the first x positive integers. The Diophantine equation

$$S_k(x) = y^n \tag{1}$$

in positive integers k, n, x, y with $n \geq 2$ has a long history, going back to Lucas [12, 13], Watson [22] and others, who considered the case $(k, n) = (2, 2)$. For details and more history we refer to the book [20] and the papers [2, 6, 7] and the references given there.

As it is well-known, in the case when (k, n) belongs to the set

$$\{(1, 2), (3, 2), (3, 4), (5, 2)\} \tag{2}$$

(1) has infinitely many solutions, which can be described easily. In 1956, Schäffer [18] proved that for any fixed (k, n) not in the set (2), equation (1) has only finitely many solutions. Schäffer's proof was not

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effective, though for certain (small) pairs (k, n) he could show that (1) has only the trivial solution $(x, y) = (1, 1)$. Further, he conjectured that for (k, n) not in the set (2), equation (1) has the only nontrivial solution $(k, n, x, y) = (2, 2, 24, 70)$. From this point on, the solutions mentioned so far will be referred to as *known solutions.*

Later, Győry, Tijdeman and Voorhoeve [8] provided an effective proof for a more general version of Schäffer's theorem, where the exponent n is also unknown. Further, under certain assumptions Pintér [16] proved that for the nontrivial solutions we have $n < ck \log(2k)$, where c is an effectively computable absolute constant. For more results concerning (1) and its various generalizations, we refer once again to the book [20] and the papers [2, 6, 7] and the references therein.

Beside the above mentioned sparse pairs (k, n) considered by Schäffer himself, Schäffer's conjecture has been verified for larger sets of the parameters involved. Jacobson, Pintér and Walsh [9] proved that the conjecture is true for $n = 2$ and even values of k with $2 \leq k \leq 58$. Later, Bennett, Győry and Pintér [2] showed that the conjecture is valid for any $n \geq 2$ for $1 \leq k \leq 11$. Recently, Pintér [17] proved that Schäffer's conjecture is also true whenever n is even with $n > 4$ and k is odd with $1 \leq k < 170$.

A common feature of all the above results is that at least one parameter in (1) is considered to be fixed, or belongs to a relatively small finite set. In this paper we prove Schäffer's conjecture under certain assumptions on x, letting the other variables k, n, y to be completely free. As far as we know, this is the first result of this type in the literature. We also mention that the assumptions imposed on x are satisfied by a positive proportion of the positive integers. In particular, for the even values of k it is sufficient to assume that $x \equiv 3, 4 \pmod 8$.

Our results mainly rely on assertions describing the exact values of $\nu_2(S_k(x))$ and $\nu_3(S_k(x))$, where $\nu_p(N)$ stands for the exponent of the prime p appearing in the prime factorization of the positive integer N. The result describing $\nu_2(S_k(x))$ is due to MacMillan and Sondow [14], while the assertion concerning $\nu_3(S_k(x))$ is new in its full generality. (The case when k is even is given by Sondow and Tsukerman [21].) Note that many assertions of a somewhat similar type are also known; see e.g. the papers [10, 15] and the references there, dealing with the Erdős-Moser conjecture concerning the solutions of the equation $S_k(x) = (x + 1)^k$, or [19] about a problem of Bednarek asking for describing those pairs (k, m) for which $S_k(x)$ divides $S_{km}(x)$ for all positive integers x; cf. also [4, 5] and the references there for certain other related problems.

Apart from this, after bounding n, we shall also use local arguments in order to solve equation (1) for fixed n. In particular, when we consider x to be also fixed, (1) is a kind of exponential-polynomial equation. Such equations are of classical and recent interest. Here we only refer to the papers [1, 3] dealing with powers having few digits, and the references there. We also mention that a result of Leitner [11] implies the solution of (1) for $n = 2$ and $x = 3$.

In the next section we give our results. Beside the already mentioned theorem yielding a positive answer for Schäffer's conjecture under certain assumptions on x, we provide upper bounds for n under more moderate conditions. Finally, we also give all solutions of equation (1) for some values of x. In the third section we give the formula for $\nu_2(S_k(x))$ from [14] and establish the formula for $\nu_3(S_k(x))$. Finally, in the last section we give the proofs of our theorems.

2. New results

Our first result provides a positive answer for the conjecture of Schäffer, under certain congruence conditions on x, letting the other three parameters k, n, y to be completely free.

Theorem 2.1. *Assume that* $x \equiv 3, 4 \pmod 8$. *Then equation* (1) *has no solutions with* $k = 1$ *or* k *even.*

Further, if one of the congruences $x \equiv h_i \pmod{m_i}$ *with* $h_i \in H_i$ $(i = 1, 2, 3, 4)$ *is also valid, where*

$$H_1 = \{2\},\ H_2 = \{5, 7\},\ H_3 = \{2, 7, 9, 14\},\ H_4 = \{18, 22\},$$

and

$$m_1 = 5,\ m_2 = 13,\ m_3 = 17,\ m_4 = 41,$$

then equation (1) *has only the known solutions.*

Remark 1. Note that it would be easy to provide further congruence conditions when the assertion of Theorem 2.1 remains valid, e.g. based upon the proof of Theorem 2.3. However, since it is clear that our present method is not capable to solve the conjecture completely, we do not want to stress this point further.

Our next result provides upper bounds for the exponent n in (1) in terms of the 2 and 3 valuations ν_2 and ν_3 of some functions of x and x, k. Recall that $\nu_p(N)$ stands for the exponent of the prime p in the prime factorization of the positive integer N.

Theorem 2.2. *i) Suppose first that* $x \equiv 0, 3 \pmod 4$*. Then for any solution* (k, n, x, y) *of equation* (1) *we have*

$$n \le \begin{cases} \nu_2(x(x+1)) - 1, & \text{if } k = 1 \text{ or } k \text{ is even}, \\ 2\nu_2(x(x+1)) - 2, & \text{if } k \ge 3 \text{ is odd}. \end{cases}$$

ii) Assume now that $x \equiv 0, 8 \pmod 9$*. Then for any solution* (k, n, x, y) *of equation* (1) *we have*

$$n \le \begin{cases} \nu_3(x(x+1)), & \text{if } k = 1, \\ \nu_3(x(x+1)(2x+1)) - 1, & \text{if } k \text{ is even}, \\ \nu_3(kx^2(x+1)^2) - 1, & \text{if } k \ge 3 \text{ is odd}. \end{cases}$$

Remark 2. Note that from the proof one can easily see that in fact n divides the expression occurring in the right hand side in the inequalities in parts i) and ii) of the theorem. We also mention that assuming that x satisfies both congruences $x \equiv 0, 3 \pmod 4$ and $x \equiv 0, 8 \pmod 9$, one can certainly combine the assertions of parts i) and ii) of Theorem 2.2.

Finally, we give the complete solution of equation (1) for values of x corresponding to part i) of Theorem 2.2. The reason why we go up to $x = 24$ is the existence of the "interesting" solution $(k, n, x, y) = (2, 2, 24, 70)$.

Theorem 2.3. *Suppose that* $x \equiv 0, 3 \pmod 4$ *and* $x < 25$*. Then equation* (1) *has only the known solutions.*

3. Formulas for $\nu_2(S_k(x))$ and $\nu_3(S_k(x))$

One of our main tools in the proofs will be precise knowledge of the values of $\nu_2(S_k(x))$ and $\nu_3(S_k(x))$. The information concerning $\nu_2(S_k(x))$ is due to MacMillan and Sondow [14], and is the following.

Lemma 3.1. *Let x be a positive integer. Then we have*

$$\nu_2(S_k(x)) = \begin{cases} \nu_2(x(x+1)) - 1, & \text{if } k = 1 \text{ or } k \text{ is even}, \\ 2\nu_2(x(x+1)) - 2, & \text{if } k \ge 3 \text{ is odd}. \end{cases}$$

The description of the value of $\nu_3(S_k(x))$ is given by the following lemma. Note that the case when k is even has been proved by Sondow and Tsukerman, see Corollary 9 in [21]. However, for the convenience of the reader, our proof covers this part of the statement, as well.

Lemma 3.2. *Let x be a positive integer. Then we have*

$$\nu_3(S_k(x)) = \begin{cases} \nu_3(x(x+1)), & \text{if } k = 1, \\ \nu_3(x(x+1)(2x+1)) - 1, & \text{if } k \text{ is even}, \\ 0, & \text{if } x \equiv 1 \pmod 3 \text{ and } k \geq 3 \text{ is odd}, \\ \nu_3(kx^2(x+1)^2) - 1, & \text{if } x \equiv 0, 2 \pmod 3 \text{ and } k \geq 3 \text{ is odd}. \end{cases}$$

Proof. Since $S_1(x) = x(x+1)/2$ and $S_2(x) = x(x+1)(2x+1)/6$ for any positive integer x, for $k = 1$ and 2 the statement is automatic. Hence from this point on we shall always assume that $k \geq 3$.

Now we shall proceed by induction on x. The statement is obvious for $x = 1$, and also for $x = 2$ if $k = 1$ or k is even. When $x = 2$ and $k \geq 3$ is odd, we can write

$$S_k(2) = 1 + (3-1)^k = 3k + \sum_{i=2}^{k} (-1)^{k-i} \binom{k}{i} 3^i.$$

By observing that

$$\nu_3\left(\binom{k}{i} 3^i\right) = \nu_3\left(\binom{k-1}{i-1}\right) + \nu_3(k) - \nu_3(i) + i > \nu_3(k) + 1 \quad (2 \leq i \leq k),$$

the statement follows in this case, as well.

Consider now the statement for some value x with $x \geq 3$, and assume that the assertion is valid for all x' with $1 \leq x' < x$ (for all positive integers k).

We distinguish two cases. Assume first that x is of the form $\varepsilon 3^\alpha$ with $\varepsilon = 1, 2$ and $\alpha \geq 1$. Now if k is even, then we have

$$S_k(3^\alpha) = \sum_{i=0}^{\frac{3^\alpha - 1}{2}} (i^k + (3^\alpha - i)^k) \equiv 2 S_k\left(\frac{3^\alpha - 1}{2}\right) \pmod{3^\alpha}$$

and

$$S_k(2 \cdot 3^\alpha) = 3^{\alpha k} + \sum_{i=0}^{3^\alpha - 1} (i^k + (2 \cdot 3^\alpha - i)^k) \equiv 2 S_k(3^\alpha - 1) \pmod{3^\alpha}$$

for $\varepsilon = 1$ and 2, respectively. Since the induction hypothesis now implies

$$\nu_3(S_k(3^\alpha)) = \nu_3\left(S_k\left(\frac{3^\alpha - 1}{2}\right)\right) = \alpha - 1$$

and

$$\nu_3(S_k(2 \cdot 3^\alpha)) = \nu_3(S_k(3^\alpha - 1)) = \alpha - 1,$$

we are done in this case. On the other hand, if k is odd then writing $k := 3^\gamma k'$ with $\gamma \geq 0$ and $3 \nmid k'$, using

$$\nu_3\left(\binom{3^\gamma}{u}3^{\alpha u}\right) \geq \gamma - \nu_3(u) + \alpha u \geq 2\alpha + \gamma \quad \text{for} \quad 2 \leq u \leq 3^\gamma$$

and

$$(3^{\alpha+\gamma} i^{3^\gamma - 1} - i^{3^\gamma})^{k'} \equiv k' 3^{\alpha+\gamma} i^{k-1} - i^k \pmod{3^{2\alpha+\gamma}}$$

by the induction hypothesis for $\varepsilon = 1$ we obtain

$$S_k(3^\alpha) = \sum_{i=0}^{\frac{3^\alpha - 1}{2}} (i^k + ((3^\alpha - i)^{3^\gamma})^{k'}) \equiv \sum_{i=0}^{\frac{3^\alpha - 1}{2}} (i^k + (3^{\alpha+\gamma} i^{3^\gamma - 1} - i^{3^\gamma})^{k'}) \equiv$$

$$\equiv \sum_{i=0}^{\frac{3^\alpha - 1}{2}} k' 3^{\alpha+\gamma} i^{k-1} \equiv \pm 3^{2\alpha+\gamma-1} \pmod{3^{2\alpha+\gamma}}$$

which proves our claim. In case of $\varepsilon = 2$ by a similar argument and with the same notation we get

$$S_k(2 \cdot 3^\alpha) = 3^{\alpha k} + \sum_{i=0}^{3^\alpha - 1} (i^k - ((2 \cdot 3^\alpha - i)^{3^\gamma})^{k'}) \equiv$$

$$\equiv \sum_{i=0}^{3^\alpha - 1} (i^k + (2 \cdot 3^{\alpha+\gamma} i^{3^\gamma - 1} - i^{3^\gamma})^{k'}) \equiv$$

$$\equiv \sum_{i=0}^{3^\alpha - 1} k' 2 \cdot 3^{\alpha+\gamma} i^{k-1} \equiv \pm 3^{2\alpha+\gamma-1} \pmod{3^{2\alpha+\gamma}}$$

and the statement follows also in this case.

Suppose next that x is *not* of the form $\varepsilon 3^\alpha$ with $\varepsilon = 1, 2$ and $\alpha \geq 1$. Then, as $x \geq 3$, by the ternary expansion of x, we can write $x = \eta 3^\beta + \varepsilon 3^\alpha$, with η a positive integer not divisible by 3, $\varepsilon = 1, 2$, and integers β and α with $\beta > \alpha \geq 0$. Then we have

$$S_k(x) = S_k(\eta 3^\beta) + \sum_{i=1}^{\varepsilon 3^\alpha} \sum_{j=0}^{k} \binom{k}{j} (\eta 3^\beta)^{k-j} i^j =$$

$$= S_k(\eta 3^\beta) + \sum_{j=0}^{k} \binom{k}{j} (\eta 3^\beta)^{k-j} S_j(\varepsilon 3^\alpha)$$

where $S_0(x) = x$. Now as

$$\nu_3\left(\binom{k}{j}\right) = \nu_3\left(\binom{k}{k-j}\right) \geq \max(\nu_3(k) - \nu_3(j), \nu_3(k) - \nu_3(k-j))$$

for $1 \le j \le k-1$, using induction one can easily see that

$$\nu_3(S_k(x)) = \nu_3(S_k(\varepsilon 3^\alpha)).$$

Hence the lemma follows. $\square$

4. Proofs of the theorems

Now we are ready to give the proofs of our theorems. We start with Theorem 2.2, since it will be used in the proofs of the other statements.

Proof of Theorem 2.2. i) Since $x \equiv 0, 3 \pmod 4$, by Lemma 3.1 we have that $\nu_2(S_k(x)) > 0$, that is, $S_k(x)$ is even. Thus if (1) holds, then $\nu_2(y) > 0$ and we have

$$n\nu_2(y) = \nu_2(y^n) = \nu_2(S_k(x)) = \begin{cases} \nu_2(x(x+1) - 1, & \text{if } k \text{ is even,} \\ 2\nu_2(x(x+1) - 2, & \text{if } k \text{ is odd,} \end{cases}$$

implying the statement in this case.

ii) As now $x \equiv 0, 8 \pmod 9$, Lemma 3.2 implies that $\nu_3(S_k(x)) > 0$. Hence (1) gives $\nu_3(y) > 0$, and noting that $x \equiv 0, 2 \pmod 3$, we have

$$n\nu_3(y) = \nu_3(y^n) = \nu_3(S_k(x)) = \begin{cases} \nu_3(x(x+1)), & \text{if } k = 1, \\ \nu_3(x(x+1)(2x+1)) - 1, & \text{if } k \text{ is even,} \\ \nu_3(kx^2(x+1)^2) - 1, & \text{if } k \ge 3 \text{ is odd,} \end{cases}$$

and the theorem is proved. $\square$

Proof of Theorem 2.1. Observe that since $x \equiv 3, 4 \pmod 8$, we have $\nu_2(x(x+1)) - 1 = 1$. Hence if $k = 1$ or k is even then by part i) of Theorem 2.2 we have $n \le 1$, which is impossible. Thus the first part of the statement follows.

So we may assume that k is odd with $k \ge 3$. Then part i) of Theorem 2.2 implies that $n = 2$. As the cases $(k, n) = (3, 2), (5, 2)$ give only known solutions, we may assume that $k \ge 7$. Then one can easily check that

$$S_k(x) \equiv y^2 \pmod{32}$$

is solvable if and only if $k \equiv 1 \pmod 8$. However, one can also readily check that in case of $x \equiv h_i \pmod{m_i}$ for any $h_i \in H_i$ $(i = 1, 2, 3, 4)$

$$S_k(x) \equiv y^2 \pmod{m_i}$$

is not solvable whenever $k \equiv 1 \pmod 8$. This implies the statement. $\square$

Proof of Theorem 2.3. Throughout the proof, we shall assume that $k \geq 9$. Since $x < 25$, the values $k < 9$ can be easily checked.

To prove the theorem for the separate values of x, first we give a bound for the exponent n using part i) of Theorem 2.2, then we handle the remaining exponents by congruences using appropriate moduli. We summarize the results of our calculations in Table 1. In fact the cases $x = 12$ and 20 are covered by Theorem 2.1, however, for the sake of completeness we include them also here. Further, note that the moduli occurring in Table 1 could certainly be "merged" into one large modulus in each case. However, we prefer the "separate" presentation because it makes the argument more transparent. Since our method is similar for each case, we only illustrate it (and also explain our notation in Table 1) through two particular instances.

First consider the case $x = 4$. Then part i) of Theorem 2.2 gives $n = 2$. Considering equation (1) modulo 16, 7 and 13 we obtain that $k \equiv 1 \pmod 4$, $k \not\equiv 1, 5 \pmod 6$ and $k \not\equiv 9 \pmod{12}$, respectively. However, combining these constraints on k yields a contradiction.

Next consider the (technically more complicated) case $x = 16$. Now part i) of Theorem 2.2 gives $n \leq 6$. Hence it is sufficient to prove the insolvability of equation (1) for $n = 2, 3, 5$. Since the congruence

$$S_k(16) \equiv y^5 \pmod{128}$$

has no solutions (under our assumption $k \geq 9$), we get that equation (1) has no solution with $n = 5$ in this case. When $n = 3$, considering our equation modulo 9 and 13, we deduce that k is odd and k is even, respectively. This of course immediately shows that $n = 3$ is impossible. Finally, if $n = 2$ then checking equation (1) modulo 512, 7 and 73 we get that $k \equiv 1 \pmod 8$, $k \equiv 3 \pmod 6$ and $k \not\equiv 9 \pmod{24}$, respectively. However, these together yield a contradiction, and our claim follows also in this case. Note that when the exponent n is not indicated in Table 1, we assume that $n = 2$.

In all the other cases a similar argument works, the details are summarized in Table 1, to be understood in a similar way as above. □

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x	bound for n	moduli and information deduced
3	2	mod 16: $k \equiv 1 \pmod 4$; mod 9: $k \equiv 3 \pmod 6$; mod 13: $k \not\equiv 9 \pmod{12}$
4	2	mod 16: $k \equiv 1 \pmod 4$; mod 7: $k \not\equiv 1, 5 \pmod 6$; mod 13: $k \not\equiv 9 \pmod{12}$
7	4	mod 32: $n \neq 3$; mod 3: k is odd; mod 5: $k \not\equiv 1 \pmod 4$; mod 128: $k \not\equiv 3 \pmod 4$
8	4	mod 32: $n \neq 3$; mod 81: $k \equiv 3 \pmod 6$; mod 128: $k \equiv 0, 1 \pmod 4$; mod 13: $k \not\equiv 9 \pmod{12}$
11	2	mod 32: $k \equiv 1 \pmod 8$; mod 9: $k \equiv 3 \pmod 6$; mod 97: $k \not\equiv 9 \pmod{24}$
12	2	mod 16: $k \equiv 1 \pmod 4$; mod 5: $k \not\equiv 1 \pmod 4$
15	6	mod 128: $n \neq 5$; mod 9: $k \equiv 0 \pmod 3$ if $n = 3$; mod 13: $k \not\equiv 3 \pmod 6$ if $n = 3$; mod 19: $k \not\equiv 0 \pmod 6$ if $n = 3$; mod 256: $k \equiv 1 \pmod 4$; mod 9: $k \not\equiv 1, 5 \pmod 6$; mod 13: $k \not\equiv 9 \pmod{12}$
16	6	mod 128: $n \neq 5$; mod 9: k is odd if $n = 3$; mod 13: k is even if $n = 3$; mod 512: $k \equiv 1 \pmod 8$; mod 7: $k \not\equiv 1, 5 \pmod 6$; mod 73: $k \not\equiv 9 \pmod{24}$
19	2	mod 32: $k \equiv 1 \pmod 8$; mod 17: $k \not\equiv 1 \pmod 8$
20	2	mod 16: $k \equiv 1 \pmod 4$; mod 13: $k \not\equiv 1 \pmod 4$
23	4	mod 32: $n \neq 3$; mod 64: $k \equiv 1 \pmod 4$; mod 7: $k \equiv 3 \pmod 6$; mod 13: $k \not\equiv 9 \pmod{12}$
24	4	mod 32: $n \neq 3$; mod 128: $k \equiv 1 \pmod 8$; mod 17: $k \not\equiv 1 \pmod 8$

TABLE 1. Data for the solution of equation (1) with $x \equiv 0, 3 \pmod 4$ and $x < 25$. If n is not indicated then $n = 2$.

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