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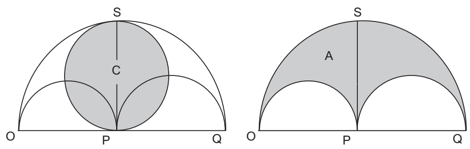
tmcs@math.klte.hu
http://tmcs.math.klte.hu
Teaching Mathematics and Computer Science

Proof without words: Four circles

Ángel Plaza

In [1] is proved the following:

Theorem. *Let O, P and Q be three points on a line, with P lying between O and Q. Semicircles are drawn on the same side of the line with diameters OP, PQ and OQ. An arbelos is the figure bounded by these three semicircles. Draw the perpendicular to OQ at P, meeting the largest semicircle at S. Then the area C of the circle with diameter PS equals the area A of the arbelos [Archimedes, Liber Assumptorum, Proposition 4].*

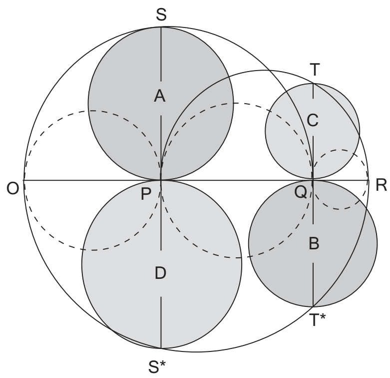


A = C

Here we prove without words the following:

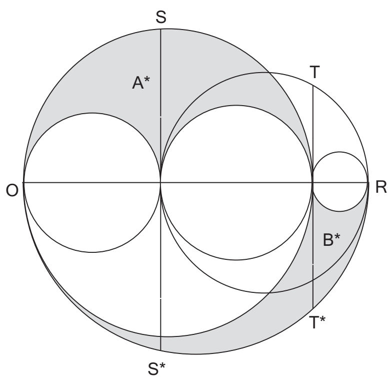
Theorem. *Let O, P, Q and R be four points on a line ordering from left to right. Semicircles are drawn on the same side of the line with with diameters OQ, PR, and semicircle with diameter OR is drawn on the other side of the line. Draw the perpendiculars to OR at P and at Q respectively, meeting the upper semicircles at S and T, and the down semicircle at S^* and T^*. Then the sum of the area of the circle with diameter PS and the area of the circle with diameter*

QT^* equals the sum of the area of the circle with diameter PS^* and the area of the circle with diameter QT.

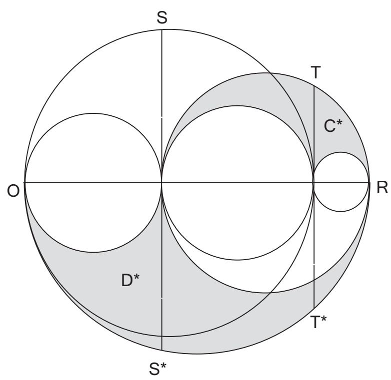


A + B = C + D

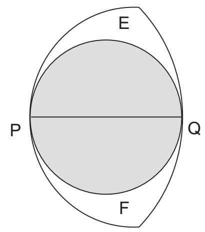
Proof.



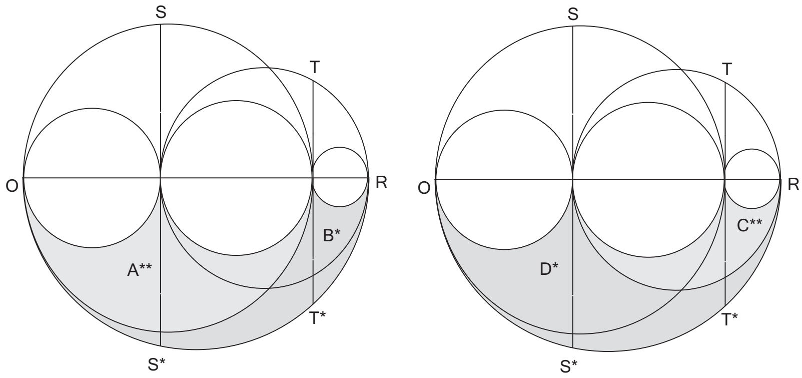
A = A* & B = B* ⟹ A + B = A*+ B*



C = C* & D = D* ⟹ C + D = C*+ D*



E = F



A* = A** & C* = C**

∵ A + B = C + D

References

[1] R. B. Nelsen, Proof without words: The area of an arbelos, *Mathematics Magazine* **75** (2002), 144.

ÁNGEL PLAZA
DEPARTMENT OF MATHEMATICS
UNIVERSITY OF LAS PALMAS DE GRAN CANARIA
EDIFICIO DE INFORMÁTICA Y MATEMÁTICAS
35017–LAS PALMAS DE GRAN CANARIA
SPAIN

E-mail: `aplaza@dmat.ulpgc.es`

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