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Analysis of A Tilting Table with Parallel Kinematics

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***Abstract*—This paper presents a workpiece moving unit with parallel kinematics, where its kinematic model is described. Based on the Chebyshev-Grübler-Kutzbach mobility equation the mobility properties of the mechanism are examined. Using the modified Chebyshev-Grübler-Kutzbach criterion the number of degrees of freedom of the workpiece tilting table is determined, after this the screw theory will be presented. As the Chebyshev-Grübler-Kutzbach criterion does not take into account the geometrical characteristics of the examined structure, using the screw theory the workpiece tilting unit will be reanalysed, to take geometrical characteristics into account in determining the degrees of freedom of the structure. Finally, the results of the two theories will be compared in the study of the given kinematic model.**

***Keywords*—machine tool, parallel kinematics, degree of freedom, mobility, screw theory**

I. INTRODUCTION

One part of the examination of the workpiece tilting unit is the determination of the degree of freedom and mobility of the structure. Several methods for determining mobility or degree of freedom have come to light over time. Among others, Chebyshev-Grübler-Kutzbach [1], [2] criterion, Screw theory [3], [4], Set theory and Lie theory [5]. The structure will be examined with the Chebyshev-Grübler-Kutzbach criterion and Screw theory. In general, testing of all mechanism is usually started by examining the degree of freedom to verify functionality. Screw theory is an effective mathematical tool for studying spatial mechanisms. One of the advantages of Screw theory is to simplify the analysis of mechanisms by geometric description of motion.

II. STRUCTURAL DESIGN OF THE WORKPIECE MOVING UNIT

The workpiece tilting unit consist of a platform, three actuators of the same design (A1, A2, A3) and a central joint (C4) (Table 1.). The platform is connected to the base via actuators and a central universal joint (Figure 1.). The actuators include two universal joint- and a prismatic joint connection in the following order: universal joint, prismatic joint, universal joint.

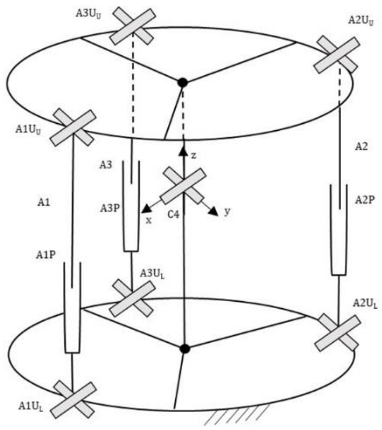

Fig. 1. Schematic view of workpiece moving table.

TABLE I.

Marking	**Marking system**			
	1. actuator	***2. actuator***	***3. actuator***	***Central connection***
Actuators	A1	A2	A3	
Universal joint for central connection				C4
Upper universal joint for a given actuator	$A1U_U$	$A2U_U$	$A3U_U$	
Prismatic connection to a given actuator	A1P	A2P	A3P	
Lower universal joint for a given actuator	$A1U_L$	$A2U_L$	$A3U_L$	

III. TRADITIONAL MOBILITY EQUATION

One of the most important factors in analysing the kinematics of mechanisms is the number of degrees of freedom. The degree of freedom of the mechanism is the number of independent parameters or inputs, which means a complete definition of the mechanism configurations. The degree of freedom of the mechanism usually depends on the number of links and the type and number of joints. The following equation gives the mobility, the degree of freedom of the mechanism [6], [7].

$$M = d(n - g - 1) + \sum_{i=1}^{g} f_i \quad (1)$$

Equation (1) is Chebyshev-Grübler-Kutzbach equation. Where:

M: degree of freedom,

d: d=6 in case of a spatial mechanism, d=3 in case of a planar mechanism,

n: number of links, including the fixed base of the mechanism,

g: the number of joints of the mechanism,

fi: the number of degrees of freedom of the ith joint.

The modified Chebyshev-Grübler-Kutzbach equation (2). will be used to calculate the mobility of the workpiece moving table.

$$M = d(n - g - 1) + \sum_{i=1}^{g} f_i - v - \zeta \quad (2)$$

Equation (2) is Modified Chebyshev-Grübler-Kutzbach equation.

Where:

M: degree of freedom,

d: d=6 in case of a spatial mechanism, d=3 in case of a planar mechanism,

n: number of links, including the fixed base of the mechanism,

g: the number of joints of the mechanism,

f_i: the number of degrees of freedom of the i-th joint,

ν: the number of parallel redundant constraints,

ζ: the degree of freedom of passive constraints.

TABLE II.

CASE	**Number of degrees of freedom for different actuator numbers.**						
	Number of actuators.	***d***	***N***	***g***	***sum fi***	***v***	***M***
1	TWO actuators	6	6	7	12	2	2
2	THREE actuators	6	8	10	17	3	2
3	FOUR actuators	6	10	13	22	4	2

Case 2. of Table II. shows the mobility of the structure of Figure 1. We performed a rapid examination of how the degree of freedom changes when the number of actuators is increased or decreased by one. Table II. shows that if we reduce (case 1.) or increase (case 3.) the number of actuators by one, the degree of freedom of the system remains unchanged according to the modified Chebyshev-Grübler-Kutzbach equation.One of the most important factors in analysing the kinematics of mechanisms is the number of degrees of freedom. The degree of freedom of the mechanism is the number of independent

IV. SCREW THEORY

The screw is a line in space with a pitch. [8],[9] The screw can be represented by its coordinates, with a pair of three-dimensional row vectors **S** and $\mathbf{S_0}$ in a Cartesian coordinate system.

$$\$ = (\mathbf{S}; \mathbf{S_0}) = (\mathbf{S}; \mathbf{p}\mathrm{x}\mathbf{S} + \mathrm{h}\mathbf{S}) =$$

$$= (\mathrm{S}_x, \mathrm{S}_y, \mathrm{S}_z, \mathrm{S}_{0x}, \mathrm{S}_{0y}, \mathrm{S}_{0z}) \quad (3)$$

Equation (3) is the equation of the screw.

Where **S** is the vector pointing along the line, that is the axis of the screw, **p** the position vector points to any point on the axis of the screw, and h is the pitch of the screw.

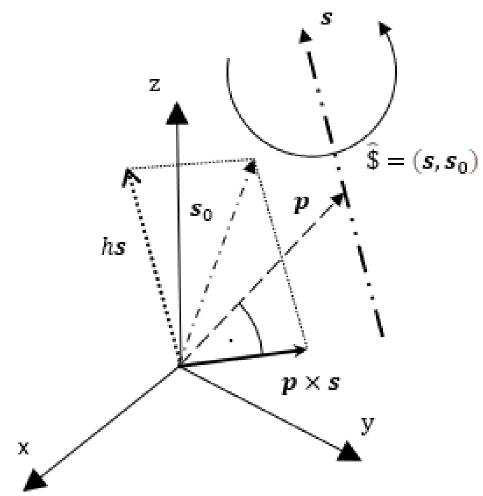

Fig. 2. The screw and its components.

Since $\mathbf{S}\cdot\mathbf{S_0} = \mathbf{S}\cdot(\mathbf{p}\mathrm{x}\mathbf{S} + \mathrm{h}\mathbf{S}) = \mathrm{h}\mathbf{S}\cdot\mathbf{S}$, the thread pitch of the screw is given by Equation (4).

$$\mathrm{h} = \frac{\mathbf{S}\cdot\mathbf{S_0}}{\mathbf{S}\cdot\mathbf{S}} \tag{4}$$

Equation (4) is the screw pitch.

Let $\boldsymbol{S} = \rho\boldsymbol{s}$, where $\rho = (\boldsymbol{S}\cdot\boldsymbol{S})^{1/2}$ is the length of the screw and s is the unit vector along the axis of the screw. Then we can write the screw expressed by a unit screw, $\hat{\$}$, as

$$\$ = \rho\hat{\$} = \rho(\boldsymbol{s}, \boldsymbol{s_0}) =$$

$$= \rho(\boldsymbol{s}, \boldsymbol{p}\times\boldsymbol{s} + h\boldsymbol{s}) =$$

$$= \rho(s_x, s_y, s_z; s_{0x}, s_{0y}, s_{0z}) \tag{5}$$

Equation (5) is screw expressed by unit screw.

Where

$$\mathbf{s}\cdot\mathbf{s} = {s_x}^2 + {s_y}^2 + {s_z}^2 = 1 \tag{6}$$

The kinematics and statics of a rigid body can be described by geometry using screw theory. In kinematics the motion of the rigid body consists of two parts, from the displacement of the screw along the axis of the unit screw, and from the rotational motion about the unit axis. If the pitch is zero, the screw turns into a simple rotation, and the coordinates of the screw become the coordinates of a line. The magnitude of a screw, as an amplitude, is the angular rotation about the axis of the screw, unless the screw is a displacement. The pure displacement is described by Equation 7. as a prismatic joint, screw with infinite pitch.

$$\$ = (\mathbf{0}; \boldsymbol{S}) = \rho(\mathbf{0}; \boldsymbol{s}) \tag{7}$$

Equation (7) is the screw is like a displacement.

In statics, forces and torques act on the rigid body, the force acts along the axis of the screw, and the torque rotates about the axis of the screw. This combination of force and torque is called screw force. The screw force is described by Equation 3., the first three component represent the force, the second three components represent the torque. The thread pitch is the ration of torque and force. If the pitch of the screw force is zero, then screw force means a force.

V. THE RECIPROCAL SCREW

It is assumed that a screw force $\$_{\mathbf{1}} = (\boldsymbol{S_1}, \boldsymbol{S_{01}})$ acts on a rigid body while it moves infinitesimally with a screw $\$_{\mathbf{2}} = (\boldsymbol{S_2}, \boldsymbol{S_{02}})$. The rigid body is in equilibrium, virtual work is zero, what the screw force does on the rigid body in the direction of the screw.

$$\delta W = [\boldsymbol{S_1}, \boldsymbol{S_{01}}][\boldsymbol{S_{02}}, \boldsymbol{S_2}]^T =$$

$$= \boldsymbol{S_1}\cdot\boldsymbol{S_{02}} + \boldsymbol{S_2}\boldsymbol{S_{01}} =$$

$$= \rho_1\rho_2(\boldsymbol{s_1}\boldsymbol{s_{02}} + \boldsymbol{s_2}\boldsymbol{s_{01}}) = \mathbf{0} \tag{8}$$

Equation (8) is reciprocity condition.

A pair of screw that satisfy Equation 8. is called reciprocal. [10],[11] Even if the screw force and the screw are interchanged, the two screws satisfy the reciprocity condition. The magnitude of the screw does not affect the reciprocity of the screw. Explaining the Equation 8. gives Equation 9.

$$\delta W = \boldsymbol{S_1}\cdot\boldsymbol{S_{02}} + \boldsymbol{S_2}\boldsymbol{S_{01}} =$$

$$= \boldsymbol{S_1}(\boldsymbol{p_2}\times\boldsymbol{S_2}) + \boldsymbol{S_2}(\boldsymbol{p_1}\times\boldsymbol{S_1}) +$$

$$+(h_1 + h_2)\boldsymbol{S_1}\cdot\boldsymbol{S_2} = \mathbf{0} \tag{9}$$

VI. DETERMINING THE DEGREE OF FREEDOM OF A TILTING TABLE WITH SCREW THEORY

The structure of the joints of the tilting table (Figure 3.) is shown in Figure 4.

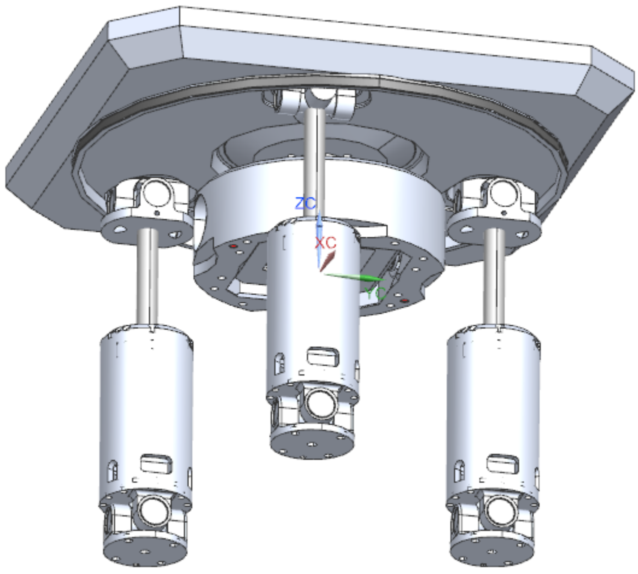

Fig. 3. The tilting table.

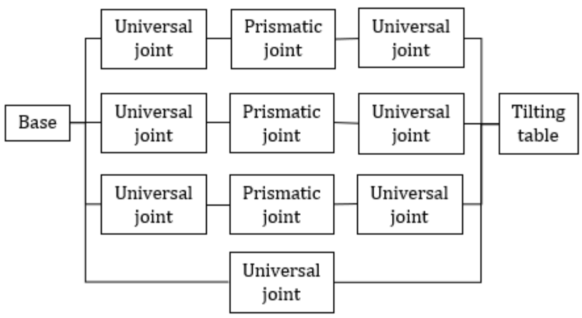

Fig. 4. Structure of the tilting table.

The structure of the tilting table is described by Equation 10., which is the twist of the titling table, which means the degree of freedom and mobility of the structure [12].

$$\$ = \left[\left(\left(\bigcup_{t=1}^{i} \$_t\right)^r\right) \cup \left(\left(\bigcup_{u=1}^{i} \$_t\right)^r\right) \cup \left(\left(\bigcup_{v=1}^{i} \$_t\right)^r\right) \cup \left(\left(\bigcup_{w=1}^{1} \$_t\right)^r\right)\right]^r \quad (10)$$

Equation (10) is the tilting table screw.

Equation 10. consistent with the process of determining the degree of freedom of parallel mechanism (figure 5.).

Determining the degree of freedom of parallel mechanisms consist of four main steps.

- Production of a screw system for the actuators.
- Production of constraint screw system for the actuators using the reciprocal condition.
- Summary of the constraint screw system of the actuators.
- Production of the platform screw using the reciprocal condition.

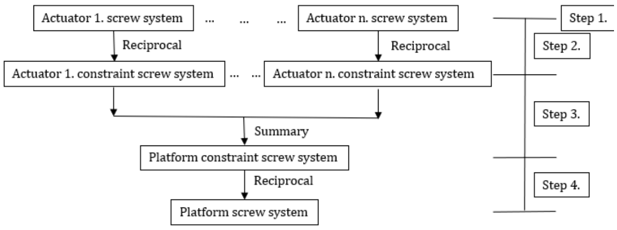

Fig. 5. The process of determining the degree of freedom of parallel mechanisms.

Following the process of determining the degree of freedom of the parallel mechanisms, we can produce the degree of freedom of the tilting table (Figure 3.). The tilting table has three actuators and a central joint (Figure 6.) and as mentioned in Chapter II., the three actuators have same structure.

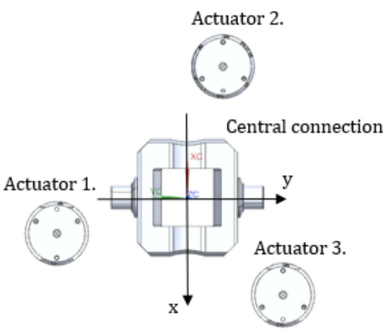

Fig. 6. Location of tilting table actuators.

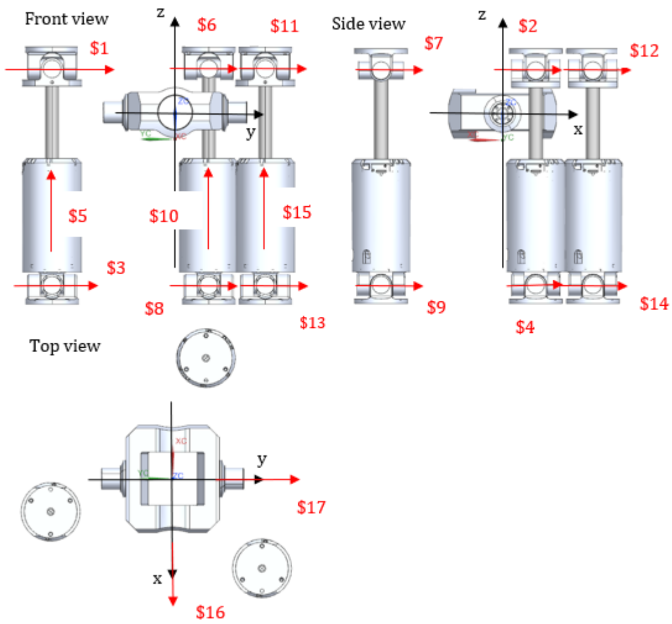

Fig. 7. Representation of the screw system of the tilting table.

- Production of a screw system for the actuators and central connection:

TABLE III.

Screw system for actuators and central connection.

Screw for the actuator 1.	***Screw for the actuator 2.***	***Screw for the actuator 3.***	***Screw for the central connection 4.***
$1=(0 1 0;-z1 0 x1) $2=(1 0 0;0 z2 y2) $3=(0 1 0;z3 0 x3) $4=(1 0 0;0 -z4 y4) $5=(0 0 0;0 0 1)	$6=(0 1 0;-z6 0 x6) $7=(1 0 0;0 z7 -y7) $8=(0 1 0;z8 0 -x8) $9=(1 0 0;0 -z9 -y9) $10=(0 0 0;0 0 1)	$11=(0 1 0;-z11 0 x11) $12=(1 0 0;0 z12 -y12) $13=(0 1 0;z13 0 x13) $14=(1 0 0;0 -z14 -y14) $15=(0 0 0;0 0 1)	$16=(1 0 0;0 0 0) $17=(0 1 0;0 0 0)

Production of constraint screw system for the actuators and central connection using the reciprocal condition.

Constrain screw for first actuator: $\$_r^1$=(1 0 0;0 0 1)

Constrain screw for second actuator: $\$_r^2$=(1 1 0;0 0 1)

Constrain screw for third actuator: $\$_r^3$=(1 0 0;0 0 1)

Constrain screw for central connection: $\$_r^4$=(1 1 1;0 0 1)

Summary of the constraint screw system of the actuators and central connection.

$$\sum_{i=1}^{4} \$_i^r = (\,4\;2\;1\;;\;0\;0\;4\,)$$

Production of the platform screw using the reciprocal condition.

$=(1 1 0;0 0 0)

The result shows that the workpiece tilting table has two degrees of freedom, it can rotate about the x and y axes.

VII. SUMMARY

Paper is presents mobility and degree of freedom determination for closed loop parallel kinematic chain. Two methods have been applied to determinate the degree of freedom of a tilting table with parallel kinematics during analysis One of the methods is the modified Chebyshev-Grübler-Kutzbach criteria which is provide mobility for analysed structure. This one is a traditional form. The other method is the Screw theory what is used for calculation of degree of freedom of structure. The Screw theory is more efficient tools for spatial mechanism. In a brief, through the Screw theory applied method of virtual work provides the degrees of freedom of the analysed system. As the results shows for two applied calculation method is in agreement for the investigation of the parallel mechanism, in Chapter III. and VI. The result calculated by the Screw theory gives a confirmation of the correctness of the result of the traditional mobility equation.

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