Effect of the Haar measure on the finite temperature effective potential of $SU(2)$ Yang-Mills theory

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Abstract

Including the Haar measure we show that the effective potential of the regularized SU(2) Yang-Mills theory has a minimum at vanishing Wilson-line $W = 0$ for strong coupling, whereas it develops two degenerate minima close to $W = \pm 1$ for weak coupling. This suggests that the non-abelian character of $SU(2)$ as contained in the Haar measure might be responsible for confinement.

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Concerning the finite temperature effective potential of $SU(2)$ Yang-Mills theory it was established by Weiss [1] that the tree level contribution of the timelike 'gluons' due to the Haar measure is cancelled by a piece of the 1-loop contribution of the longitudinal gluons. This finding has a far reaching importance for the understanding of confinement. If the non-abelian character of $SU(2)$ is the crucial factor for confinement one would expect the Haar measure to play an important role. Consequently, it should induce a contribution to the effective potential dominating at large distances. On the other hand if it does not contribute at all (as suggested by the results of Weiss) this supports the idea propagated e.g. by Gribov [2] that only the strength of the interaction is crucial.

The 2–loop contributions of the order g^2 to the effective potential have been established in the perturbative regime [3], i.e. for non-vanishing Wilson line $W = \pm 1$. Allowing for arbitrary values of W, we show now explicitly that the cancellation observed by Weiss does not hold in the order g^2, i.e. the 1-loop contribution of the Haar measure is not cancelled by the 2-loop contribution due to the usual Yang-Mills self-interaction. Thus the Haar measure contributes to the effective potential and leads to a minimum at $W = 0$ for sufficiently strong coupling. A similar result has been obtained for $SU(2)$ lattice gauge theory in Ref. [4]. For weak coupling we shall recover the results obtained in Ref. [1].

General arguments were given that a deconfining phase transition occurs with increasing temperature if the Yang-Mills theory is confining at zero temperature [5, 6]. It was also shown that lattice gauge theory does not confine static quarks if the Haar measure is replaced by the Euclidean one [7]. Here we show that the minimum found at $W=0$ corresponds to confinement. How the continuum limit can be taken and to what extent the non-trivial minimum does survive remains of course an open question which can only be answered by renormalization group analysis being out of the scope of the present letter.

Similarly to [1] we use a time independent, diagonal gauge, and periodic boundary conditions for the spatial components of the vector potential in the 'time' direction. We allow for the non-vanishing vacuum expectation value $v \equiv \langle a^{-1}\beta g A^{03}\rangle \equiv a^{-1}\beta C$ ($A^{\mu a}$ the gluon vector potential, μ and a the Lorentz and $SU(2)$ colour indices, respectively). Throughout this paper we use the notations of Ref. [1] and use the same cut-off regularization. The UV cut-off Λ is interpreted in terms of the lattice spacing a used for the definition of the path integral via $a^{-3} = (2\pi)^{-3}\int_0^\Lambda d^3k$ with $\Lambda = (6\pi^2)^{1/3}a^{-1}$ and d^3k the volume element in 3-momentum space. The same spacing a is assumed in 'time' direction.

Let us write for the timelike gluon field $A^{03}(\vec{x}) = C/g + \delta\phi(\vec{x})$ and for the spatial components $A^{ia}(\vec{x},t)$ ($i=1,2,3$; $a=1,2,3$). Expanding the Haar measure in powers of the fluctuation $\delta\phi(\vec{x})$ and including all terms up to the order g^2, we obtain the tree level effective action, $S_{eff} = S_0 + S_1 + S_2$ where

$$\begin{aligned} S_0 &= \frac{1}{2a^4}\int_0^\beta dt \int d^3x \left[a^2(\nabla\delta\phi)^2 + (a\partial^0 A^i + C\hat{3}\times A^i)^2 + \frac{1}{2}a^2(\partial^i A^j - \partial^j A^i)^2\right] \\ &\quad -\frac{1}{a^3}\int d^3x \ln(1-\cos v) + \frac{1}{2a^3}\frac{g^2a^{-2}\beta^2}{1-\cos v}\int d^3x(\delta\phi)^2 \qquad (1) \end{aligned}$$

($\hat{3}$ is the unit vector in the direction 3 of colour space), S_1 and S_2 are the cubic and quartic self-interaction terms, respectively. The last term of S_0 proportional to g^2 was neglected in Ref. [1], as well as the self-interaction S_1 and S_2. In order to calculate the finite temperature effective potential, we determine the partition function Z treating S_1 and S_2 as perturbations of the 'free' theory defined by S_0. The free propagator of the field $\delta\phi$ acquires now the mass term $a^{-1}g^2\beta(1-\cos v)^{-1} \equiv M^2 a^2$ due to the Haar measure generated quadratic self-interaction.

The effective potential is given in terms of the one-particle irreducible part of the logarithm of the partition function, $V_{eff}(\beta; v) = -(\beta V)^{-1} \ln Z_{\mathrm{1PI}} = V_0(\beta; v) + \Delta V(\beta; v)$. The effective potential for the 'free' theory $V_0(\beta; v) = V_W(\beta; v) + V_H(\beta; v)$ consists of the term $V_W(\beta; v)$ of the order g^0 found by Weiss [1] and the term $V_H(\beta; v)$ of the order g^2 generated by the Haar measure:

$$V_H(\beta; v) = \frac{1}{\beta V} \int \frac{d^3k}{(2\pi)^3} \ln\left(\frac{g^2 a^{-1}\beta}{a^2k^2(1-\cos v)} + 1 \right) \approx \frac{g^2}{a^4 \alpha_0 \sin^2(v/2)} + \mathcal{O}(g^4) \qquad (2)$$

with $\alpha_0 = (6\pi^2)^{1/2}$ for $\beta/a \to \infty$. The 2-loop contribution $\Delta V(\beta; v)$ was obtained by carrying out the Matsubara sums with the standard techniques of finite temperature field theory [10]. UV-divergences have been removed by subtracting from each 2-loop diagram those with the same structure but a single loop taken in the limit $\beta \to \infty$ in all possible ways. The IR momentum cut-off μ was chosen in an UV cut-off (a) dependent way by requiring that the free energy of the perturbative vacuum does not depend on μ. Then the free energy of the perturbative vacuum turned out to be a temperature independent constant which was subtracted. As a result of this choice of the IR cut-off μ all terms of $\Delta V(\beta, v)$ depending on μ vanish in the $\beta/a \to \infty$ limit and we obtain:

$$\begin{aligned} \Delta V(\beta, v) \;&=\; \frac{g^2}{24\beta^4}\left\{1+\frac{8}{\pi^2}\left|\sin\frac{v}{2}\right|\left(\left|\sin\frac{v}{2}\right|-\frac{2\pi}{3}\right)+\frac{1}{\pi^4}\left(\frac{4\pi^2}{3}+G(v)\right)G(v)\right. \\ &\left.+\frac{21}{4\pi^4 e^2}\frac{\sin^2 v}{(\cosh 1-\cos v)^2}(1-e^{-1})(2-5e^{-1})\right\} \end{aligned} \tag{3}$$

with

$$G(v) \;=\; \frac{2}{e}\left(\frac{\cos v - e^{-1}}{1+e^{-2}-2e^{-1}\cos v}-\frac{1}{1-e^{-1}}\right) \tag{4}$$

and e the basis of the natural logarithm. Our conclusion is that the 2-loop contribution ΔV due to the non-abelian gluon self-interaction $S_1 + S_2$ does not cancel the 1-loop contribution V_H due to the Haar-measure. We also see that the v-dependent terms of ΔV vanish for vanishing background field $v = 0$. For $v = 0$ we obtained $\Delta V_{v=0} = \frac{1}{24}g^2\beta^{-4}$ which is identical to the 2-loop contribution of gluons and ghosts found in [3]. It needs further investigation how our results would be modified by using lattice regularized propagators for the calculation of the loop integrals.

Let us express the effective potential in terms of the vacuum expectation value of the Wilson line operator,

$$W(\vec{x}) \;=\; \left\langle \frac{1}{2}\mathrm{tr}\left(\prod_{j=1}^{\beta/a}\exp\left\{ig\phi(\vec{x})\tau^3/2\right\}\right)\right\rangle \approx \cos\left(\frac{v}{2}\right) \equiv W \tag{5}$$

for $R = g^2(\beta/a)^4$ fixed and $\beta/a \to \infty$ ($|W| \leq 1$). For $\beta/a \to \infty$ we can neglect the 2-loop contribution ΔV as compared to the Haar measure term V_H. It turns out that

$v = 0$, i.e. $W = 0$ is a minimum of the effective potential for $R > R_c \equiv 2\alpha_0/3$, i.e. for strong coupling and it turns over in a maximum for $R < R_c$, i.e. for weak coupling. In the latter case we obtain two minima positioned at $W \approx \pm(1-(R/8R_c)^{1/2})$ for $R \ll R_c$, and at $W \approx \pm(1-(R/R_c))^{1/2}$ for $R \approx R_c$ $(R < R_c)$. Thus the effective potential in the approximation used exhibits the features of a second order phase transition with the order parameter $|W|$ vanishing smoothly for $R \to R_c$ (see Fig. 1). The infinite values of V_H at $W = \pm 1$ occur due to the fact that the expansion used for the Haar-measure potential is not valid for $v = 0$ and $\pm 2\pi$. Unluckily the critical temperature T_c defined by $g^2(aT_c)^{-4} = R_c = 5.1$ decreases rather slowly with decreasing coupling ($aT_c \sim 0.94$ to 0.71 for $4/g^2 \sim 1$ to 3) and does not agree quantitatively with the numerical results of lattice gauge simulations [4]. The relation between these two approaches has to be further investigated.

It is rather intriguing to extract information on the behaviour of the correlator of Wilson-lines. For strong coupling the vacuum is characterized by $v = \pi$ and the free energy of a static quark-antiquark pair increases linearly with their separation distance in leading order:

$$F_{q\bar{q}} \;=\; -\beta^{-1} \ln\langle \mathcal{W}(\vec{x})\mathcal{W}(\vec{y})\rangle = \beta^{-1} M|\vec{x}-\vec{y}| + \beta^{-1} \ln \frac{|\vec{x}-\vec{y}|}{a} + \text{ const.} \qquad (6)$$

This corresponds to the string tension $\kappa = \beta^{-1}M = \beta^{-2}[aR/(2\beta)]^{1/2}$. For weak coupling $R \ll 1$ the degenerate vacua are at $v = \epsilon$ and $\pm(2\pi - \epsilon)$ with $0 < \epsilon^2 = (8R/R_c)^{1/2} \ll 1$ and the correlator of the Wilson lines the Debye-screened Coulomb law $F_{q\bar{q}} \sim e^{-M|\vec{x}-\vec{y}|}/|\vec{x}-\vec{y}|$ with the screening length $1/M = \beta(\beta/a)^{1/2}\sqrt{2/R} \to \infty$ for $R \to 0$.

Summarizing, we established that the finite temperature effective potential of the regularized $SU(2)$ Yang-Mills theory exhibits rather different qualitative behaviour in the strong coupling and weak coupling limits. For strong coupling ($R \gg R_c$) the effective potential has a single minimum for vanishing Wilson-line $W = 0$ ($v = \pi$), and the vacuum state is confining. On the other hand, there are degenerate minima at $W \to 1$ ($v \to 0$) and $W \to -1$ ($v \to \pm 2\pi$) and a maximum at $W = 0$ for weak coupling ($R \to 0$). These minima correspond to a vacuum state in which static quarks are not confined.

The results obtained hint to the possibility that the Haar measure induced vertices could be responsible for confinement. If proven this would be of fundamental importance. For a proof, however, a resummation of the contributions of all induced vertices to the effective potential is needed. Furthermore an investigation of the renormalization group trajectories is needed to establish the effective potential in the continuum limit.

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Figure Caption

Fig. 1 The order parameter $|W|$ of the deconfining phase transition vs. R/R_c.